Math 233 - October 23, 2009
Solutions

- How do you compute work when the vector field is a conservative field?

1. Let $r(t) = (t+1, 2t-4)$ for $-1 \leq t \leq 5$.
   (a) Find a parametrization for the reverse path $r^{-}(t) = r(-1-t) = (-t-2, -t)$, $-2 \leq t \leq 1$. (If possible, try to make r and $^{-}$ have the same domain.)
   (b) Find the work done along r and r^{-} by the vector field $F = (x+y, x+2y)$.

$$\int_r F \cdot dr = 54$$
$$\int_{r^-} F \cdot dr = -54$$

2. Let $r(t) = (t-1, t+1)$ for $-2 \leq t \leq 1$.
   (a) Find a parametrization for the reverse path $r^{-}(t) = r(-1-t) = (-t-2, -t)$, $-2 \leq t \leq 1$. (If possible, try to make r and $^{-}$ have the same domain.)
   (b) Find the work done along r and r^{-} by the vector field $F = (x^2-1, xy)$.

$$\int_r F \cdot dr = 6$$
$$\int_{r^-} F \cdot dr = -6$$

3. Find potential functions for the following vector fields
   (a) $F_1 = (2x - yz + y^2, 2xy - xz, -xy)$
   $\phi_1 = x^2 + xy^2 - xyz$
   (b) $F_2 = (y, x + 2ye^z, y^2e^z)$
   $\phi_2 = xy + y^2e^z$
   (c) $F_3 = (-2xz/(1+x^2+y^2)^2, -2yz/(1+x^2+y^2)^2, 1/(1+x^2+y^2))$
   $\phi_3 = \frac{z}{1+x^2+y^2}$

Lecture Problems

4. Let $r(t) = (t^3 - t^2 + t, 2t^4 - 3t^3 + 1, 2 - t^2)$ for $-1 \leq t \leq 1$. Find the work done by the fields in Problem 1 over this curve.
   (a) $\int_r F_1 \cdot dr = \phi_1(1,0,1) - \phi_1(-3,6,1) = 82$
   (b) $\int_r F_2 \cdot dr = \phi_2(1,0,1) - \phi_2(-3,6,1) = 18 - 36e$

(c) $\int_r F_3 \cdot dr = \phi_3(1,0,1) - \phi_3(-3,6,1) = \frac{11}{23}$

5. True or false. What changes would you have to make to the false statments to make them true?

   (a) If the vector field F is conservative then the work done along any path only depends on the endpoints of that path.

   Solution: True

   (b) If the curl of the vector field F is $(0,0,0)$ then the work done along any path only depends on the endpoints of that path.

   Solution: False. Would be true if we restricted the domain of F to a rectangle.

   (c) If the work done by the vector field F around every loop is 0 then the vector field is conservative.

   Solution: True

   (d) If the work done by the vector field F along any path depends only on the end points of the path then the vector field is conservative.

   Solution: True