Math 233 - October 22, 2009
Solutions

- If you compute work from point A to point B, does the actual path travelled change the amount of work?

Remember that work done by a vector field over a curve is given by

$$\int_r F \cdot dr = \int_a^b F(r(t)) \cdot r'(t)\, dt$$

1. Let $F = (3x^2 + y, x + y)$. Compute the work that F does over the paths below.

   (Note that the starting point of each path is $(1, 0)$ and the ending point of each path is $(-1, 0)$. You should make sure you draw all paths.)

   (a) $r(t) = (-t, 0)$, $-1 \leq t \leq 1$. $W = -2$

   (b) $r(t) = (\cos t, \sin t)$, $0 \leq t \leq \pi$. $W = -2$

   (c) $r(t)$ is the path that first connects $(1, 0)$ to $(0, 1)$ by a line segment, then connects $(0, 1)$ to $(-1, 0)$ by a line segment. $W = -2$

   (d) $r(t)$ is the path that first connects $(1, 0)$ to $(0, -1)$ by a line segment, then connects $(0, -1)$ to $(-1, 0)$ by a line segment. $W = -2$

2. Oreo Cookie Question 1: Is the work done between two points independent of the path taken between the two points?

Lecture Problems

3. Let $F = (-y, x)$. Compute the work that F does over the paths below.

   (Note that the starting point of each path is $(1, 0)$ and the ending point of each path is $(-1, 0)$. You should make sure you draw all paths.)

   (a) $r(t) = (-t, 0)$, $-1 \leq t \leq 1$. $W = 0$

   (b) $r(t) = (\cos t, \sin t)$, $0 \leq t \leq \pi$. $W = \pi$

   (c) $r(t)$ is the path that first connects $(1, 0)$ to $(0, 1)$ by a line segment, then connects $(0, 1)$ to $(-1, 0)$ by a line segment. $W = 1 + 1 = 2$

   (d) $r(t)$ is the path that first connects $(1, 0)$ to $(0, -1)$ by a line segment, then connects $(0, -1)$ to $(-1, 0)$ by a line segment. $W = -1 - 1 = -2$

4. Oreo Cookie Question 1: Is the work done between two points independent of the path taken between the two points?

5. Oreo Cookie Question 2: For some of the paths you had to come up with the parametrization of the curve. Would the work change if you changed the parametrization?

6. Let $G = \left(\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \right)$.

   (Note: some of these integrals are slightly messy, but they should be doable.)

(a) $r(t) = (-t, 0)$, $-1 \leq t \leq 1$. $W =$ DNE!

(b) $r(t) = (\cos t, \sin t)$, $0 \leq t \leq \pi$. $W = \pi$

(c) $r(t)$ is the path that first connects $(1, 0)$ to $(0, 1)$ by a line segment, then connects $(0, 1)$ to $(-1, 0)$ by a line segment. $W = \pi/2 + \pi/2 = \pi$

(d) $r(t)$ is the path that first connects $(1, 0)$ to $(0, -1)$ by a line segment, then connects $(0, -1)$ to $(-1, 0)$ by a line segment. $W = \ -\pi/2 + -\pi/2 = -\pi$

7. Reverse path.

(a) Let

$$r(t) = (t, t^2) \qquad 1 \leq t \leq 3$$

What are the starting and ending points for $r(t)$?

Solution: Start: $(1, 1)$. End: $(3, 9)$.

(b) Find a parametrization for the *reverse path* of $r(t)$–the path that traces out the same path but with the starting and ending points swapped.

$$r^-(t) = r(4 - t) = (4 - t, (4 - t)^2) \qquad 1 \leq t \leq 3$$

Did you use the same interval of definition?

It should be clear that this curve has the same trace as $r(t)$. Checking the start and end points should convince you it is the reverse path.

(c) Let $F = (x, x + y)$. Compute the work done by F over the path r and its reverse path r^-.

$$\int_r F \cdot dr = \frac{184}{3}$$

$$\int_{r^-} F \cdot dr^- = -\frac{184}{3}$$