If you compute work from point $A$ to point $B$, does the actual path travelled change the amount of work?

Remember that work done by a vector field over a curve is given by

\[ \int_{r} F \cdot dr = \int_{a}^{b} F(r(t)) \cdot r'(t) \, dt \]

1. Let $F = (3x^2 + y, x + y)$. Compute the work that $F$ does over the paths below.

   (Note that the starting point of each path is $(1,0)$ and the ending point of each path is $(-1,0)$. You should make sure you draw all paths.)

   (a) $r(t) = (-t, 0), -1 \leq t \leq 1$. $W = -2$

   (b) $r(t) = (\cos t, \sin t), 0 \leq t \leq \pi$. $W = -2$

   (c) $r(t)$ is the path that first connects $(1,0)$ to $(0,1)$ by a line segment, then connects $(0,1)$ to $(-1,0)$ by a line segment. $W = -2$

   (d) $r(t)$ is the path that first connects $(1,0)$ to $(0,-1)$ by a line segment, then connects $(0,-1)$ to $(-1,0)$ by a line segment. $W = -2$

2. Oreo Cookie Question 1: Is the work done between two points independent of the path taken between the two points?

Lecture Problems

3. Let $F = (-y, x)$. Compute the work that $F$ does over the paths below.

   (Note that the starting point of each path is $(1,0)$ and the ending point of each path is $(-1,0)$. You should make sure you draw all paths.)

   (a) $r(t) = (-t, 0), -1 \leq t \leq 1$. $W = 0$

   (b) $r(t) = (\cos t, \sin t), 0 \leq t \leq \pi$. $W = \pi$

   (c) $r(t)$ is the path that first connects $(1,0)$ to $(0,1)$ by a line segment, then connects $(0,1)$ to $(-1,0)$ by a line segment. $W = 1 + 1 = 2$

   (d) $r(t)$ is the path that first connects $(1,0)$ to $(0,-1)$ by a line segment, then connects $(0,-1)$ to $(-1,0)$ by a line segment. $W = -1 - 1 = -2$

4. Oreo Cookie Question 1: Is the work done between two points independent of the path taken between the two points?

5. Oreo Cookie Question 2: For some of the paths you had to come up with the parametrization of the curve. Would the work change if you changed the parametrization?

6. Let $G = \left( \frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \right)$.

   (Note: some of these integrals are slightly messy, but they should be doable.)
(a) \( r(t) = (-t, 0), -1 \leq t \leq 1. \) \( W = \text{DNE!} \)

(b) \( r(t) = (\cos t, \sin t), 0 \leq t \leq \pi. \) \( W = \pi \)

(c) \( r(t) \) is the path that first connects \((1, 0)\) to \((0, 1)\) by a line segment, then connects \((0, 1)\) to \((-1, 0)\) by a line segment. \( W = \pi/2 + \pi/2 = \pi \)

(d) \( r(t) \) is the path that first connects \((1, 0)\) to \((0, -1)\) by a line segment, then connects \((0, -1)\) to \((-1, 0)\) by a line segment. \( W = -\pi/2 - \pi/2 = -\pi \)

7. Reverse path.

(a) Let
\[
r(t) = (t, t^2) \quad 1 \leq t \leq 3
\]
What are the starting and ending points for \( r(t) \)?

Solution: Start: (1, 1). End: (3, 9).

(b) Find a parametrization for the reverse path of \( r(t) \)–the path that traces out the same path but with the starting and ending points swapped.

\[
r^-(t) = r(4 - t) = (4 - t, (4 - t)^2) \quad 1 \leq t \leq 3
\]

Did you use the same interval of definition?

It should be clear that this curve has the same trace as \( r(t) \). Checking the start and end points should convince you it is the reverse path.

(c) Let \( F = (x, x + y) \). Compute the work done by \( F \) over the path \( r \) and its reverse path \( r^- \).

\[
\int_r F \cdot dr = \frac{184}{3}
\]

\[
\int_{r^-} F \cdot dr^- = -\frac{184}{3}
\]