Math 233 - October 20, 2009

- Test Review!

1. Find the maximum rate of change among all possible directions for the function $f(x, y) = x^2y^3 + 2x^4y$ at the point $(-1, 1)$.
2. Let $f(x, y) = \ln(x^2 + y^3)$.
   (a) Find a linearization of f near the point $(2, 1)$
   (b) Find the equation of the tangent plane to the graph at $(2, 1, \ln 5)$.
   (c) Find a unit vector in the direction in which f increases most rapidly from the point $(2, 1)$.
3. Approximate $f(8.04, 11.97)$ for a function $f(x, y)$ for which you know $f(8, 12) = 3$, $f_x(8, 12) = -1$ and $f_y(8, 12) = 2$.
4. Find the equation of the tangent plane at $(2, 1, 3)$ to $z = \sqrt{20 - x^2 - 7y^2}$.
5. find the directional derivative of $f(x, y) = x^2 + xy$ at $(1, 2)$ in the direction of the vector $(1, 1)$
6. Find the equation of the tangent plane at the point $(1, 2, 0)$ to the surface $x^3 + 2y^2 + z^2 - 2xy - 2yz = 5$.
7. Let $f(x, y)$ be differentiable and defined on the set $A = \{(x, y)|x^2+4y^2 < 1\}$. Suppose $f_x(0, 0) = 0$ and $f_y(0, 0) = 0$. Which must be true
   (a) f has a local maximum on A
   (b) f has a local minimum on A
   (c) f has a local extremum on A
   (d) f has a absolute maximum on A
   (e) f has a absolute minimum on A
8. Find and classify all critical points to $f(x, y) = 8y^3 + 12x^2 - 24xy$.
9. Find the max of $f(x, y) = 3y + 4x$ on the circle $x^2 + y^2 = 1$.
10. Find the maximum rate of change of $f(x, y) = x^3y^4$ at the point $(1, 1)$.
11. Find the maximum of $f(x, y, z) = 2x + y + z$ for points satisfying $x^2 + y^2 + z^2 \leq 6$.
12. Find the minimum of $f(x, y) = 2x^2 + 3y^2$ subject to $x^2 + y^2 = 4$.
13. Find and classify all critical points of $f(x, y) = x^3 + 3xy + 3y^2/2$
14. Let $f(x, y) = xy$. Find all extrema on the set $4x^2 + 9y^2 \leq 36$.
15. Find and classify all critical points of $f(x, y) = 3xy^2 + x^3 - 3y^2 - 3x^2 + 5$.

16. Let a, b, c be fixed positive numbers. Find the positive numbers x, y, z such that $ax+by+cz = 99$ and xyz is maximal. (You may be able to do this without calculus, but obviously you will need to know how to do it using calculus.)

17. Let $f(x, y) = x/y^2$. Make two approximations of $f(3.2, 1.1)$. First use the linerization at the point $(4, 1)$ and then use the second order Taylor polynomial at the point $(4, 1)$.

18. Let $f(x, y) = x^3 + y^4 - \ln(x + y - 2)$. Make two approximations of $f(1.2, 1.9)$. First use the linerization at the point $(1, 2)$ and then use the second order Taylor polynomial at the point $(1, 2)$.

19. True/False
    (a) The set in $\mathbb{R}^2$ defined by $xy = 1$ is closed.
    (b) The set in $\mathbb{R}^2$ defined by $xy = 1$ is bounded.
    (c) If f is differentiable then the tangent plane at any critical is parallel to the xy plane.
    (d) A continuous function on a closed domain has a maximum and minimum on that closed set.
    (e) A continuous function on a bounded domain has a maximum and minimum on that bounded set.
    (f) All vector fields have potential functions.
    (g) All gradient vector fields have potential functions.
    (h) If the curl of a vector field is $(0, 0, 0)$ then the vector field has a potential function.

20. Find the curl of $F = (e^x \sin y, e^x \cos y, z)$

21. Let $f(x, y, z) = x^2 \cos(z/x) + y/\ln(x - \tan z)$.
    Find a potential function for the vector field $F(x, y) = \nabla f(x, y)$.

22. Let $F = (2xy, x^2 + 1)$. Find a potential function $f(x, y)$ for F and find $f(1, 1) - f(0, 0)$.

23. Let $F = (e^y, xe^y, (z + 1)e^z)$. Let f be a potential function for F and find $f(1, 1, 1) - f(0, 0, 0)$.

24. Find the curl of the vector field $F = (z^2, zy, x^2 - z)$

25. Find the absolute maximum of $f(x, y) = x^2-2xy+2y$ on the rectangle $\{(x, y)|0 \leq x \leq 3 \text{ and } 0 \leq y \leq 2\}$.

26. Find and classify all criticals points of the function $f(x, y) = x^4 + y^4 - 4xy + 2$.

27. Use Lagrange multipliers to find the minimum distance from the curve $y = x^2 - 1$ to the origin.

28. Find a potential function for the vector field $F = (x, y + z, z^2)$.

29. Find a potential function for the vector field $F = (e^x, 2yz, y^2)$.

30. Find a potential function for the vector field $F = (ye^z, ze^y, xe^z)$.

31. Find the divergence of $F = (5x^2 + 3yz, 7y^2 + 2xz, 3z^2 + 3xy)$