Test Review!

1. Find the maximum rate of change among all possible directions for the function \( f(x, y) = x^2y^3 + 2x^4y \) at the point \((-1, 1)\).

2. Let \( f(x, y) = \ln(x^2 + y^3) \).
   (a) Find a linearization of \( f \) near the point \((2, 1)\)
   (b) Find the equation of the tangent plane to the graph at \((2, 1, \ln 5)\).
   (c) Find a unit vector in the direction in which \( f \) increases most rapidly from the point \((2, 1)\).

3. Approximate \( f(8.04, 11.97) \) for a function \( f(x, y) \) for which you know \( f(8, 12) = 3 \), \( f_x(8, 12) = -1 \) and \( f_y(8, 12) = 2 \).

4. Find the equation of the tangent plane at \((2, 1, 3)\) to \( z = \sqrt{20 - x^2 - 7y^2} \).

5. Find the directional derivative of \( f(x, y) = x^2 + xy \) at \((1, 2)\) in the direction of the vector \((1, 1)\).

6. Find the equation of the tangent plane at the point \((1, 2, 0)\) to the surface \( x^3 + 2y^2 + z^2 - 2xy - 2yz = 5 \).

7. Let \( f(x, y) \) be differentiable and defined on the set \( A = \{(x, y) | x^2 + 4y^2 < 1 \} \). Suppose \( f_x(0, 0) = 0 \) and \( f_y(0, 0) = 0 \). Which must be true
   (a) \( f \) has a local maximum on \( A \)
   (b) \( f \) has a local minimum on \( A \)
   (c) \( f \) has a local extremum on \( A \)
   (d) \( f \) has a absolute maximum on \( A \)
   (e) \( f \) has a absolute minimum on \( A \)

8. Find and classify all critical points to \( f(x, y) = 8y^3 + 12x^2 - 24xy \).

9. Find the max of \( f(x, y) = 3y + 4x \) on the circle \( x^2 + y^2 = 1 \).

10. Find the maximum rate of change of \( f(x, y) = x^3y^4 \) at the point \((1, 1)\).

11. Find the maximum of \( f(x, y, z) = 2x + y + z \) for points satisfying \( x^2 + y^2 + z^2 \leq 6 \).

12. Find the minimum of \( f(x, y) = 2x^2 + 3y^2 \) subject to \( x^2 + y^2 = 4 \).

13. Find and classify all critical points of \( f(x, y) = x^3 + 3xy + 3y^2/2 \).

14. Let \( f(x, y) = xy \). Find all extrema on the set \( 4x^2 + 9y^2 \leq 36 \).

15. Find and classify all critical points of \( f(x, y) = 3xy^2 + x^3 - 3y^2 - 3x^2 + 5 \).
16. Let $a, b, c$ be fixed positive numbers. Find the positive numbers $x, y, z$ such that $ax+by+cz = 99$ and $xyz$ is maximal. (You may be able to do this without calculus, but obviously you will need to know how to do it using calculus.)

17. Let $f(x, y) = x/y^2$. Make two approximations of $f(3, 2, 1)$. First use the linerization at the point $(4, 1)$ and then use the second order Taylor polynomial at the point $(4, 1)$.

18. Let $f(x, y) = x^3 + y^4 - \ln(x + y - 2)$. Make two approximations of $f(1, 2, 1.9)$. First use the linerization at the point $(1, 2)$ and then use the second order Taylor polynomial at the point $(1, 2)$.

19. True/False
   
   (a) The set in $\mathbb{R}^2$ defined by $xy = 1$ is closed.
   (b) The set in $\mathbb{R}^2$ defined by $xy = 1$ is bounded.
   (c) If $f$ is differentiable then the tangent plane at any critical is parallel to the $xy$ plane.
   (d) A continuous function on a closed domain has a maximum and minimum on that closed set.
   (e) A continuous function on a bounded domain has a maximum and minimum on that bounded set.
   (f) All vector fields have potential functions.
   (g) All gradient vector fields have potential functions.
   (h) If the curl of a vector field is $(0, 0, 0)$ then the vector field has a potential function.

20. Find the curl of $F = (e^x \sin y, e^x \cos y, z)$

21. Let $f(x, y, z) = x^2 \cos(z/x) + y/\ln(x - \tan z)$. Find a potential function for the vector field $F(x, y) = \nabla f(x, y)$.

22. Let $F = (2xy, x^2 + 1)$. Find a potential function $f(x, y)$ for $F$ and find $f(1, 1) - f(0, 0)$.

23. Let $F = (e^y, xe^y, (z + 1)e^z)$. Let $f$ be a potential function for $F$ and find $f(1, 1, 1) - f(0, 0, 0)$.

24. Find the curl of the vector field $F = (z^2, 2y + x^2 - z)$

25. Find the absolute maximum of $f(x, y) = x^2 - 2xy + 2y$ on the rectangle $\{(x, y) | 0 \leq x \leq 3$ and $0 \leq y \leq 2\}$.

26. Find and classify all critical points of the function $f(x, y) = x^4 + y^4 - 4xy + 2$.

27. Use Lagrange multipliers to find the minimum distance from the curve $y = x^2 - 1$ to the origin.

28. Find a potential function for the vector field $F = (x, y + z, z^2)$.

29. Find a potential function for the vector field $F = (e^x, 2yz, y^2)$.

30. Find a potential function for the vector field $F = (ye^z, ze^y, xe^z)$.

31. Find the divergence of $F = (5x^2 + 3yz, 7y^2 + 2xz, 3z^2 + 3xy)$.