Math 233 - October 19, 2009
Solutions

• How do you compute the work a vector field does along a curve?

1. Let \( f(x, y) \) be a function such that \( f(0, 0) = 0 \) and \( \nabla f = (2x - 3y, 2y - 3x) \). What is \( f(1, 1) \)?

Solution: \( -1 \)

2. The function \( f(x, y) = x^3y + 12x^2 - 8y \) has one critical point \((a, b)\). Find \( f(a, b) \) and determine if it is a maximum, minimum or saddle point.

Solution: \( f(2, -4) = 48 \), saddle point.

3. Find the directional derivative of \( f(x, y, z) = x^2y + 2z \) in the direction of \( A = (1, 0, -1) \) at the point \((2, 1, 0)\).

Solution: \( D_A f(2, 1, 0) = u \cdot \nabla f = \sqrt{2} \)

Lecture Problems

4. Compute the work done by the vector field along the curve.

(a) \( F = (x + y, x^2) \), \( r(t) = (t, t^2) \) for \( 0 \leq t \leq 1 \).

\[ \int r \cdot F \, dr = \frac{4}{3} \]

(b) \( F = (-y, x) \), \( r(t) = (t, t^2) \) for \( 0 \leq t \leq 1 \).

\[ \int r \cdot F \, dr = \frac{4}{3} \]

(c) \( F = (-y^2, x) \), \( r(t) = (t + 1, 2t) \) for \( -2 \leq t \leq 2 \).

\[ \int r \cdot F \, dr = -\frac{40}{3} \]

5. Compute the curve integrals

(a) Integrate along the curve line connecting \((0, 0)\) to \((-1, 4)\).

\[ \int (x + y) \, dx + (x - y) \, dy = -\frac{23}{2} \]

(b) Integrate along the two line segments, first from \((0, 0)\) to \((-1, 0)\) then to \((-1, 4)\).

(Note: this means you should do two integrals.)

\[ \int (x + y) \, dx + (x - y) \, dy = \frac{1}{2} + (-12) = -\frac{23}{2} \]
(c) Integrate along the curve \( r(t) = (t^2, t^3), \ 0 \leq t \leq 3/2. \)

\[
\int_{r} (x^2 - y^2) \, dx + 2xy \, dy = \frac{8505}{512}
\]

6. Oreo Cookie Question

Can you find a vector field \( F \) and two paths, \( r_1 \) and \( r_2 \), each connecting the same two points, such that

\[
\int_{r_1} F \cdot dr_1 \neq \int_{r_2} F \cdot dr_2
\]

Either prove that this cannot be done or find a vector field and two paths that give different work integrals.