Math 233 - October 19, 2009
Solutions

- How do you compute the work a vector field does along a curve?

1. Let $f(x, y)$ be a function such that $f(0, 0) = 0$ and $\nabla f = (2x - 3y, 2y - 3x)$. What is $f(1, 1)$?

   Solution: -1

2. The function $f(x, y) = x^3y + 12x^2 - 8y$ has one critical point (a, b). Find $f(a, b)$ and determine if it is a maximum, minimum or saddle point.

   Solution: $f(2, -4) = 48$, saddle point.

3. Find the directional derivative of $f(x, y, z) = x^2y + 2z$ in the direction of $A = (1, 0, -1)$ at the point $(2, 1, 0)$.

   Solution: $D_A f(2, 1, 0) = u \cdot \nabla f = \sqrt{2}$

Lecture Problems

4. Compute the work done by the vector field along the curve.

   (a) $F = (x + y, x^2)$, $r(t) = (t, t^2)$ for $0 \leq t \leq 1$.

   $$\int_r F \cdot dr = \frac{4}{3}$$

   (b) $F = (-y, x)$, $r(t) = (t, t^2)$ for $0 \leq t \leq 1$.

   $$\int_r F \cdot dr = \frac{4}{3}$$

   (c) $F = (-y^2, x)$, $r(t) = (t + 1, 2t)$ for $-2 \leq t \leq 2$.

   $$\int_r F \cdot dr = -\frac{40}{3}$$

5. Compute the curve integrals

   (a) Integrate along the curve line connecting $(0, 0)$ to $(-1, 4)$.

   $$\int_r (x + y)\, dx + (x - y)\, dy = -\frac{23}{2}$$

   (b) Integrate along the two line segments, first from $(0, 0)$ to $(-1, 0)$ then to $(-1, 4)$. (Note: this means you should do two integrals.)

   $$\int_r (x + y)\, dx + (x - y)\, dy = \frac{1}{2} + (-12) = -\frac{23}{2}$$

(c) Integrate along the curve $r(t) = (t^2, t^3)$, $0 \le t \le 3/2$.

$$\int_r (x^2 - y^2)\, dx + 2xy\, dy = \frac{8505}{512}$$

6. Oreo Cookie Question

   Can you find a vector field F and two paths, r_1 and r_2, each connecting the same two points, such that

$$\int_{r_1} F \cdot dr_1 \neq \int_{r_2} F \cdot dr_2$$

   Either prove that this can not be done or find a vector field and two paths that give different work integrals.