Math 233 - October 19, 2009

- How do you compute the work a vector field does along a curve?

1. Let $f(x, y)$ be a function such that $f(0, 0) = 0$ and $\nabla f = (2x - 3y, 2y - 3x)$. What is $f(1, 1)$?
2. The function $f(x, y) = x^3y + 12x^2 - 8y$ has one critical point (a, b). Find $f(a, b)$ and determine if it is a maximum, minimum or saddle point.
3. Find the directional derivative of $f(x, y, z) = x^2y + 2z$ in the direction of $A = (1, 0, -1)$ at the point $(2, 1, 0)$.

1. Let $f(x, y)$ be a function such that $f(0, 0) = 0$ and $\nabla f = (2x - 3y, 2y - 3x)$. What is $f(1, 1)$?
   Solution: -1
2. The function $f(x, y) = x^3y + 12x^2 - 8y$ has one critical point (a, b). Find $f(a, b)$ and determine if it is a maximum, minimum or saddle point.
   Solution: $f(2, -4) = 48$, saddle point.
3. Find the directional derivative of $f(x, y, z) = x^2y + 2z$ in the direction of $A = (1, 0, -1)$ at the point $(2, 1, 0)$.
   Solution: $D_A f(2, 1, 0) = u \cdot \nabla f = \sqrt{2}$

Lecture Problems

4. Compute the work done by the vector field along the curve.

   (a) $F = (x + y, x^2)$, $r(t) = (t, t^2)$ for $0 \leq t \leq 1$.

   $$\int_r F \cdot dr =$$

   (b) $F = (-y, x)$, $r(t) = (t, t^2)$ for $0 \leq t \leq 1$.

   $$\int_r F \cdot dr =$$

   (c) $F = (-y^2, x)$, $r(t) = (t + 1, 2t)$ for $-2 \leq t \leq 2$.

   $$\int_r F \cdot dr =$$

Lecture Problems

4. Compute the work done by the vector field along the curve.

   (a) $F = (x + y, x^2)$, $r(t) = (t, t^2)$ for $0 \leq t \leq 1$.

   $$\int_r F \cdot dr = \frac{4}{3}$$

   (b) $F = (-y, x)$, $r(t) = (t, t^2)$ for $0 \leq t \leq 1$.

   $$\int_r F \cdot dr = \frac{4}{3}$$

   (c) $F = (-y^2, x)$, $r(t) = (t + 1, 2t)$ for $-2 \leq t \leq 2$.

   $$\int_r F \cdot dr = -\frac{40}{3}$$

5. Compute the curve integrals

(a) Integrate along the curve line connecting $(0,0)$ to $(-1,4)$.

$$\int_r (x+y)\ dx + (x-y)\ dy =$$

(b) Integrate along the two line segments, first from $(0,0)$ to $(-1,0)$ then to $(-1,4)$.
(Note: this means you should do two integrals.)

$$\int_r (x+y)\ dx + (x-y)\ dy =$$

(c) Integrate along the curve $r(t) = (t^2, t^3)$, $0 \leq t \leq 3/2$.

$$\int_r (x^2 - y^2)\ dx + 2xy\ dy =$$

5. Compute the curve integrals

(a) Integrate along the curve line connecting $(0,0)$ to $(-1,4)$.

$$\int_r (x+y)\,dx + (x-y)\,dy = -\frac{23}{2}$$

(b) Integrate along the two line segments, first from $(0,0)$ to $(-1,0)$ then to $(-1,4)$.
(Note: this means you should do two integrals.)

$$\int_r (x+y)\,dx + (x-y)\,dy = \frac{1}{2} + (-12) = -\frac{23}{2}$$

(c) Integrate along the curve $r(t) = (t^2, t^3)$, $0 \le t \le 3/2$.

$$\int_r (x^2 - y^2)\,dx + 2xy\,dy = \frac{8505}{512}$$

6. Oreo Cookie Question

Can you find a vector field F and two paths, r_1 and r_2, each connecting the same two points, such that

$$\int_{r_1} F \cdot dr_1 \neq \int_{r_2} F \cdot dr_2$$

Either prove that this can not be done or find a vector field and two paths that give different work integrals.