

Math 233 - October 15, 2009

Solutions

Exam 2: Oct 21, Sections 4.2 - 7.3

- Tangent planes
- Directional derivative
- Critical points
- Extrema on a closed and bounded domain
- Lagrange multipliers
- Taylor polynomials (up to second order)
- Second derivative test for extrema
- Potential functions: existence, finding

$$G = \left(-\frac{y}{x^2 + y^2}, \frac{x}{x^2 + y^2} \right)$$

- A potential function for G in the right half-plane ($x > 0$) is

$$\phi = \arctan \frac{y}{x}$$

Can we extend this potential function to be defined continuously on $\mathbb{R}^2$?

1. Find $\nabla\phi$ for

$$\phi = \arccos \frac{x}{\sqrt{x^2 + y^2}}$$

Two hints: $\frac{d}{dt} \arccos t = -\frac{1}{\sqrt{1-t^2}}$

and remember that $\sqrt{y^2} = |y|$ and not y .

$$\nabla\phi = \left(-\frac{y^2}{|y|(x^2 + y^2)}, \frac{xy}{|y|(x^2 + y^2)} \right)$$

Thus, for $y \geq 0$ we have ϕ is a potential function for G .

Lecture Problems

2. For each vector field, determine the domain, find a potential function or explain why a potential function does not exist.

- (a) $F = (1/(x - y), 1/(y - x))$
 $\phi = \ln |x - y|$
- (b) $F = (-2x/(x^2 + y^2)^2, -2y/(x^2 + y^2)^2)$
 $\phi = 1/(x^2 + y^2)$
- (c) $F = (1/y, -x/y^2)$
 $\phi = x/y$
- (d) $F = (-1/(x^2y), -1/xy^2)$
 $\phi = 1/(xy)$
- (e) $F(x, y, z) = (y, x + x/(y^2 + z^2), -y/(y^2 + z^2))$
 $\phi = xy + \arctan(y/z)$
- (f) $F(x, y, z, w) = (-y/(x^2 + y^2), x/(x^2 + y^2), 1/w^2, -2z/w^3)$
 $\phi = z/w^2 + \arctan(y/x)$

3. Find the line integrals

- (a) $F = (x + y, x^2), r(t) = (t, t^2)$ for $0 \leq t \leq 1$.

$$\int_r F \cdot dr = \frac{4}{3}$$

- (b) $F = (-y, x), r(t) = (t, t^2)$ for $0 \leq t \leq 1$.

$$\int_r F \cdot dr = \frac{4}{3}$$

- (c) $F = (-y^2, x), r(t) = (t + 1, 2t)$ for $-2 \leq t \leq 2$.

$$\int_r F \cdot dr = -\frac{40}{3}$$