Math 233 - October 15, 2009

Exam 2: Oct 21, Sections 4.2 - 7.3

- Tangent planes
- Directional derivative
- Critical points
- Extrema on a closed and bounded domain
- Lagrange multipliers
- Taylor polynomials (up to second order)
- Second derivative test for extrema
- Potential functions: existence, finding

$$G = \left(-\frac{y}{x^2 + y^2}, \frac{x}{x^2 + y^2} \right)$$

- A potential function for G in the right half-plane ($x > 0$) is

  $$\phi = \arctan \frac{y}{x}$$

  Can we extend this potential function to be defined continuously on $\mathbb{R}^2$?

1. Find $\nabla\phi$ for

$$\phi = \arccos \frac{x}{\sqrt{x^2 + y^2}}$$

Two hints: $\frac{d}{dt}\arccos t = -\frac{1}{\sqrt{1-t^2}}$
and remember that $\sqrt{y^2} = |y|$ and not y.

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$$\nabla\phi = \left(-\frac{y^2}{|y|(x^2 + y^2)}, \frac{xy}{|y|(x^2 + y^2)}\right)$$

Thus, for $y \geq 0$ we have ϕ is a potential function for G.

Lecture Problems

2. For each vector field, determine the domain, find a potential function or explain why a potential function does not exist.

   (a) $F = (1/(x-y), 1/(y-x))$
   $\phi =$

   (b) $F = (-2x/(x^2+y^2)^2, -2y/(x^2+y^2))$
   $\phi =$

   (c) $F = (1/y, -x/y^2)$
   $\phi =$

   (d) $F = (-1/(x^2y), -1/xy^2)$
   $\phi =$

   (e) $F(x,y,z) = (y, x + x/(y^2+z^2), -y/(y^2+z^2))$
   $\phi =$

   (f) $F(x,y,z,w) = (-y/(x^2+y^2), x/(x^2+y^2), 1/w^2, -2z/w^3)$
   $\phi =$

Lecture Problems

2. For each vector field, determine the domain, find a potential function or explain why a potential function does not exist.

   (a) $F = (1/(x-y), 1/(y-x))$
   $\phi = \ln|x-y|$

   (b) $F = (-2x/(x^2+y^2)^2, -2y/(x^2+y^2))$
   $\phi = 1/(x^2+y^2)$

   (c) $F = (1/y, -x/y^2)$
   $\phi = x/y$

   (d) $F = (-1/(x^2y), -1/xy^2)$
   $\phi = 1/(xy)$

   (e) $F(x,y,z) = (y, x + x/(y^2+z^2), -y/(y^2+z^2))$
   $\phi = xy + \arctan(y/z)$

   (f) $F(x,y,z,w) = (-y/(x^2+y^2), x/(x^2+y^2), 1/w^2, -2z/w^3)$
   $\phi = z/w^2 + \arctan(y/x)$

3. Find the line integrals

(a) $F = (x + y, x^2)$, $r(t) = (t, t^2)$ for $0 \le t \le 1$.

$$\int_r F \cdot dr =$$

(b) $F = (-y, x)$, $r(t) = (t, t^2)$ for $0 \le t \le 1$.

$$\int_r F \cdot dr =$$

(c) $F = (-y^2, x)$, $r(t) = (t + 1, 2t)$ for $-2 \le t \le 2$.

$$\int_r F \cdot dr =$$

3. Find the line integrals

(a) $F = (x + y, x^2)$, $r(t) = (t, t^2)$ for $0 \leq t \leq 1$.

$$\int_r F \cdot dr = \frac{4}{3}$$

(b) $F = (-y, x)$, $r(t) = (t, t^2)$ for $0 \leq t \leq 1$.

$$\int_r F \cdot dr = \frac{4}{3}$$

(c) $F = (-y^2, x)$, $r(t) = (t + 1, 2t)$ for $-2 \leq t \leq 2$.

$$\int_r F \cdot dr = -\frac{40}{3}$$