Math 233 - October 15, 2009

Exam 2: Oct 21, Sections 4.2 - 7.3

- Tangent planes
- Directional derivative
- Critical points
- Extrema on a closed and bounded domain
- Lagrange multipliers
- Taylor polynomials (up to second order)
- Second derivative test for extrema
- Potential functions: existence, finding

\[
G = \left( -\frac{y}{x^2 + y^2}, \frac{x}{x^2 + y^2} \right)
\]

- A potential function for \( G \) in the right half-plane \( (x > 0) \) is
  \[
  \phi = \arctan \frac{y}{x}
  \]
  Can we extend this potential function to be defined continuously on \( \mathbb{R}^2 \)?

1. Find \( \nabla \phi \) for

   \[
   \phi = \arccos \frac{x}{\sqrt{x^2 + y^2}}
   \]

   Two hints: \( \frac{d}{dt} \arccos t = -\frac{1}{\sqrt{1-t^2}} \)
   and remember that \( \sqrt{y^2} = |y| \) and not \( y \).
Lecture Problems

2. For each vector field, determine the domain, find a potential function or explain why a potential function does not exist.

(a) \( F = (1/(x - y), 1/(y - x)) \)
\( \phi = \)

(b) \( F = (-2x/(x^2 + y^2)^2, -2y/(x^2 + y^2)) \)
\( \phi = \)

(c) \( F = (1/y, -x/y^2) \)
\( \phi = \)

(d) \( F = (-1/(x^2y), -1/xy^2) \)
\( \phi = \)

(e) \( F(x, y, z) = (y, x + x/(y^2 + z^2), -y/(y^2 + z^2)) \)
\( \phi = \)

(f) \( F(x, y, z, w) = (-y/(x^2 + y^2), x/(x^2 + y^2), 1/w^2, -2z/w^3) \)
\( \phi = \)

3. Find the line integrals

(a) \( F = (x + y, x^2), r(t) = (t, t^2) \) for \( 0 \leq t \leq 1. \)
\[ \int_r F \cdot dr = \]

(b) \( F = (-y, x), r(t) = (t, t^2) \) for \( 0 \leq t \leq 1. \)
\[ \int_r F \cdot dr = \]

(c) \( F = (-y^2, x), r(t) = (t + 1, 2t) \) for \( -2 \leq t \leq 2. \)
\[ \int_r F \cdot dr = \]