Math 233 - October 15, 2009

Exam 2: Oct 21, Sections 4.2 - 7.3

- Tangent planes
- Directional derivative
- Critical points
- Extrema on a closed and bounded domain
- Lagrange multipliers
- Taylor polynomials (up to second order)
- Second derivative test for extrema
- Potential functions: existence, finding

$$G = \left(-\frac{y}{x^2+y^2}, \frac{x}{x^2+y^2}\right)$$

- A potential function for G in the right half-plane ($x > 0$) is

$$\phi = \arctan\frac{y}{x}$$

Can we extend this potential function to be defined continuously on $\mathbb{R}^2$?

1. Find $\nabla\phi$ for

$$\phi = \arccos\frac{x}{\sqrt{x^2+y^2}}$$

Two hints: $\frac{d}{dt}\arccos t = -\frac{1}{\sqrt{1-t^2}}$
and remember that $\sqrt{y^2} = |y|$ and not y.

Lecture Problems

2. For each vector field, determine the domain, find a potential function or explain why a potential function does not exist.

   (a) $F = (1/(x-y), 1/(y-x))$
   $\phi =$

   (b) $F = (-2x/(x^2+y^2)^2, -2y/(x^2+y^2))$
   $\phi =$

   (c) $F = (1/y, -x/y^2)$
   $\phi =$

   (d) $F = (-1/(x^2y), -1/xy^2)$
   $\phi =$

   (e) $F(x, y, z) = (y, x + x/(y^2+z^2), -y/(y^2+z^2))$
   $\phi =$

   (f) $F(x, y, z, w) = (-y/(x^2+y^2), x/(x^2+y^2), 1/w^2, -2z/w^3)$
   $\phi =$

3. Find the line integrals

   (a) $F = (x+y, x^2)$, $r(t) = (t, t^2)$ for $0 \leq t \leq 1$.

   $$\int_r F \cdot dr =$$

   (b) $F = (-y, x)$, $r(t) = (t, t^2)$ for $0 \leq t \leq 1$.

   $$\int_r F \cdot dr =$$

   (c) $F = (-y^2, x)$, $r(t) = (t+1, 2t)$ for $-2 \leq t \leq 2$.

   $$\int_r F \cdot dr =$$