Math 233 - October 12, 2009
Solutions

- Do all vector fields have potential functions?
- How can you tell if a vector field has a potential function?

1. Compute (Hint, use the substitution $u = y/x$)

$$\int \frac{x\,dy}{x^2+y^2} = \arctan\left(\frac{y}{x}\right) + C$$

2. Let $F = (f_1, f_2, f_3)$ be a vector field where f_1, f_2 and f_3 are arbitrary functions. What is

$$\begin{aligned}\text{curl}F = \nabla \times F &= \begin{vmatrix} E_1 & E_2 & E_3 \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f_1 & f_2 & f_3 \end{vmatrix} \\ &= \left(\frac{\partial f_3}{\partial y} - \frac{\partial f_2}{\partial z}, \frac{\partial f_1}{\partial z} - \frac{\partial f_3}{\partial x}, \frac{\partial f_2}{\partial x} - \frac{\partial f_1}{\partial y}\right)\end{aligned}$$

3. For the following vector fields, compute the curl of the vector field and then, if possible, find a potential function for the vector field.

   (a) $F = (yz, xz, xy)$, curl $F = (0, 0, 0)$
   $\phi = xyz$

   (b) $F = (x, y, x)$, curl $F = (0, -1, 0)$
   $\phi =$ DNE

   (c) $F = (2x, 2y, 1)$, curl $F = (0, 0, 0)$
   $\phi = x^2 + y^2 + z$

   (d) $F = (-y, z, x)$, curl $F = (-1, -1, 1)$
   $\phi =$ DNE

   (e) $F = (y, x - 2z, 2z - 2y)$, curl $F = (0, 0, 0)$
   $\phi = xy - 2yz + z^2$

4. Compute the curl of the vector field

$$F = \left(-\frac{y}{x^2+y^2}, \frac{x}{x^2+y^2}, 0\right)$$

Solution: This is just a computation and should give curl $F = (0, 0, 0)$.

Lecture Problems

5. Determine if the following sets are connected

(a) $A = \{(x,y,z) | x^2 + y^2 \leq 1\} \subset \mathbb{R}^3$
Solution: A is connected.

(b) $A = \{(x,y,z) | x^2 + y^2 = 1\} \subset \mathbb{R}^3$
Solution: A is connected.

(c) $A = \{(x,y,z) | x \neq 0\} \subset \mathbb{R}^3$
Solution: A is not connected.

(d) $A = \{(x,y) | x \neq 0\} \subset \mathbb{R}^2$
Solution: A is not connected.

(e) $A = \{(x,y,z) | x \neq 0\} \subset \mathbb{R}^3$
Solution: A is not connected.

(f) $A = \{(x,y) | x \neq 0 \text{ and } y \neq 0\} \subset \mathbb{R}^2$
Solution: A is connected.

(g) $A = \{(x,y,z) | x \neq 0 \text{ and } y \neq 0\} \subset \mathbb{R}^3$
Solution: A is connected.