How do you find a potential function?

Does every vector field have a potential function?
1. How many zeros are at the end of the decimal representation for the number

\[ 100! = 100 \times 99 \times 98 \times 97 \times \cdots \times 2 \times 1 \]
Lecture Problems

2. Compute the curl and divergence of the vector fields
   (a) \( F = (x, y, z), \) curl \( F = \)
       \( \text{div } F = \)
   (b) \( F = (x^2, y^2, z^2), \) curl \( F = \)
       \( \text{div } F = \)
   (c) \( F = (y, x, 1), \) curl \( F = \)
       \( \text{div } F = \)
   (d) \( F = (-y, x, 1), \) curl \( F = \)
       \( \text{div } F = \)
   (e) \( F = (-yz, xy, z), \) curl \( F = \)
       \( \text{div } F = \)
Lecture Problems

2. Compute the curl and divergence of the vector fields
   
   (a) \( F = (x, y, z) \), curl \( F = (0, 0, 0) \)
       \( \text{div} \ F = 3 \)
   
   (b) \( F = (x^2, y^2, z^2) \), curl \( F = (0, 0, 0) \)
       \( \text{div} \ F = 2x + 2y + 2z \)
   
   (c) \( F = (y, x, 1) \), curl \( F = (0, 0, 0) \)
       \( \text{div} \ F = 0 \)
   
   (d) \( F = (-y, x, 1) \), curl \( F = (0, 0, 2) \)
       \( \text{div} \ F = 0 \)
   
   (e) \( F = (-yz, xy, z) \), curl \( F = (0, -y, z + y) \)
       \( \text{div} \ F = x + 1 \)
3. Find a potential function, if possible, for the given vector fields.

(a) \( F(x, y) = (y^2, 2xy - 1) \). Find \( \phi \) such that \( \nabla \phi = F \).
\[ \phi = \]

(b) \( F(x, y) = (2xy, x^2 - \sin y) \). Find \( \phi \) such that \( \nabla \phi = F \).
\[ \phi = \]

(c) \( F(x, y) = (1 + ye^x, e^x) \). Find \( \phi \) such that \( \nabla \phi = F \).
\[ \phi = \]

(d) \( F(x, y) = (1 + \sin(2y - x), -2\sin(2y - x)) \). Find \( \phi \) such that \( \nabla \phi = F \).
\[ \phi = \]
3. Find a potential function, if possible, for the given vector fields.

(a) $F(x, y) = (y^2, 2xy - 1)$. Find $\phi$ such that $\nabla\phi = F$.
   $\phi = xy^2 - y$

(b) $F(x, y) = (2xy, x^2 - \sin y)$. Find $\phi$ such that $\nabla\phi = F$.
   $\phi = x^2y + \cos y$

(c) $F(x, y) = (1 + ye^x, e^x)$. Find $\phi$ such that $\nabla\phi = F$.
   $\phi = x + ye^x$

(d) $F(x, y) = (1 + \sin(2y - x), -2\sin(2y - x))$. Find $\phi$ such that $\nabla\phi = F$.
   $\phi = x + \cos(2y - x)$
4. Find a potential function, if possible, for the given vector fields.

(a) \( F(x, y, z) = (x, y, z) \). Find \( \phi \) such that \( \nabla \phi = F \).
\[ \phi = \]

(b) \( F(x, y, z) = (2xy, x^2, 1) \). Find \( \phi \) such that \( \nabla \phi = F \).
\[ \phi = \]

(c) \( F(x, y, z) = (2xy + z + 1, 4z^2 + x^2, 6yz + x) \). Find \( \phi \) such that \( \nabla \phi = F \).
\[ \phi = \]

(d) \( F(x, y, z) = (y \cos xy, x \cos xy, 1) \). Find \( \phi \) such that \( \nabla \phi = F \).
\[ \phi = \]

(e) \( F(x, y, z) = (1/(y + 2z), −x/(y + 2z)^2, −2z/(y + 2z)^2) \). Find \( \phi \) such that \( \nabla \phi = F \).
\[ \phi = \]

(f) \( F(x, y, z) = (z + 2x, 1, x) \). Find \( \phi \) such that \( \nabla \phi = F \).
\[ \phi = \]
4. Find a potential function, if possible, for the given vector fields.

(a) \( F(x, y, z) = (x, y, z) \). Find \( \phi \) such that \( \nabla \phi = F \).
\[
\phi = \frac{x^2}{2} + \frac{y^2}{2} + \frac{z^2}{2}
\]

(b) \( F(x, y, z) = (2xy, x^2, 1) \). Find \( \phi \) such that \( \nabla \phi = F \).
\[
\phi = x^2y + z
\]

(c) \( F(x, y, z) = (2xy + z + 1, 4z^2 + x^2, 6yz + x) \). Find \( \phi \) such that \( \nabla \phi = F \).
\[
\phi = x + x^2y + xz + 3yz^2
\]

(d) \( F(x, y, z) = (y \cos xy, x \cos xy, 1) \). Find \( \phi \) such that \( \nabla \phi = F \).
\[
\phi = z + \sin xy
\]

(e) \( F(x, y, z) = (1/(y + 2z), -x/(y + 2z)^2, -2z/(y + 2z)^2) \). Find \( \phi \) such that \( \nabla \phi = F \).
\[
\phi = x/(y + 2z)
\]

(f) \( F(x, y, z) = (z + 2x, 1, x) \). Find \( \phi \) such that \( \nabla \phi = F \).
\[
\phi = x^2 + y + xz
\]
5. Find a potential function, if possible, for the given vector fields.

(a) $F(x, y, z) = (y, x, 0)$. Find $\phi$ such that $\nabla \phi = F$.
   $\phi = $

(b) $F(x, y, z) = (y, -x, 0)$. Find $\phi$ such that $\nabla \phi = F$.
   $\phi = $

(c) $F(x, y, z) = (x, y, y)$. Find $\phi$ such that $\nabla \phi = F$.
   $\phi = $

(d) $F(x, y, z) = (x, x, yz)$. Find $\phi$ such that $\nabla \phi = F$.
   $\phi = $
5. Find a potential function, if possible, for the given vector fields.

(a) $F(x, y, z) = (y, x, 0)$. Find $\phi$ such that $\nabla \phi = F$.
   $\phi = xy$

(b) $F(x, y, z) = (y, -x, 0)$. Find $\phi$ such that $\nabla \phi = F$.
   $\phi = \text{DNE}$

(c) $F(x, y, z) = (x, y, y)$. Find $\phi$ such that $\nabla \phi = F$.
   $\phi = \text{DNE}$

(d) $F(x, y, z) = (x, x, yz)$. Find $\phi$ such that $\nabla \phi = F$.
   $\phi = \text{DNE}$