Math 233 - October 9, 2009

- How do you find a potential function?
- Does every vector field have a potential function?

1. How many zeros are at the end of the decimal representation for the number

$$100! = 100 \times 99 \times 98 \times 97 \times \cdots \times 2 \times 1$$

Lecture Problems

2. Compute the curl and divergence of the vector fields
   (a) $F = (x, y, z)$, $\operatorname{curl} F =$
   $\operatorname{div} F =$
   (b) $F = (x^2, y^2, z^2)$, $\operatorname{curl} F =$
   $\operatorname{div} F =$
   (c) $F = (y, x, 1)$, $\operatorname{curl} F =$
   $\operatorname{div} F =$
   (d) $F = (-y, x, 1)$, $\operatorname{curl} F =$
   $\operatorname{div} F =$
   (e) $F = (-yz, xy, z)$, $\operatorname{curl} F =$
   $\operatorname{div} F =$

Lecture Problems

2. Compute the curl and divergence of the vector fields
   (a) $F = (x, y, z)$, $\operatorname{curl} F = (0, 0, 0)$
   $\operatorname{div} F = 3$
   (b) $F = (x^2, y^2, z^2)$, $\operatorname{curl} F = (0, 0, 0)$
   $\operatorname{div} F = 2x + 2y + 2z$
   (c) $F = (y, x, 1)$, $\operatorname{curl} F = (0, 0, 0)$
   $\operatorname{div} F = 0$
   (d) $F = (-y, x, 1)$, $\operatorname{curl} F = (0, 0, 2)$
   $\operatorname{div} F = 0$
   (e) $F = (-yz, xy, z)$, $\operatorname{curl} F = (0, -y, z + y)$
   $\operatorname{div} F = x + 1$

3. Find a potential function, if possible, for the given vector fields.
   (a) $F(x, y) = (y^2, 2xy - 1)$. Find ϕ such that $\nabla\phi = F$.
   $\phi =$
   (b) $F(x, y) = (2xy, x^2 - \sin y)$. Find ϕ such that $\nabla\phi = F$.
   $\phi =$
   (c) $F(x, y) = (1 + ye^x, e^x)$. Find ϕ such that $\nabla\phi = F$.
   $\phi =$
   (d) $F(x, y) = (1 + \sin(2y - x), -2\sin(2y - x))$. Find ϕ such that $\nabla\phi = F$.
   $\phi =$

3. Find a potential function, if possible, for the given vector fields.
   (a) $F(x, y) = (y^2, 2xy - 1)$. Find ϕ such that $\nabla\phi = F$.
   $\phi = xy^2 - y$
   (b) $F(x, y) = (2xy, x^2 - \sin y)$. Find ϕ such that $\nabla\phi = F$.
   $\phi = x^2y + \cos y$
   (c) $F(x, y) = (1 + ye^x, e^x)$. Find ϕ such that $\nabla\phi = F$.
   $\phi = x + ye^x$
   (d) $F(x, y) = (1 + \sin(2y - x), -2\sin(2y - x))$. Find ϕ such that $\nabla\phi = F$.
   $\phi = x + \cos(2y - x)$

4. Find a potential function, if possible, for the given vector fields.
   (a) $F(x, y, z) = (x, y, z)$. Find ϕ such that $\nabla\phi = F$.
   $\phi =$
   (b) $F(x, y, z) = (2xy, x^2, 1)$. Find ϕ such that $\nabla\phi = F$.
   $\phi =$
   (c) $F(x, y, z) = (2xy + z + 1, 4z^2 + x^2, 6yz + x)$. Find ϕ such that $\nabla\phi = F$.
   $\phi =$
   (d) $F(x, y, z) = (y \cos xy, x \cos xy, 1)$. Find ϕ such that $\nabla\phi = F$.
   $\phi =$
   (e) $F(x, y, z) = (1/(y + 2z), -x/(y + 2z)^2, -2z/(y + 2z)^2)$. Find ϕ such that $\nabla\phi = F$.
   $\phi =$
   (f) $F(x, y, z) = (z + 2x, 1, x)$. Find ϕ such that $\nabla\phi = F$.
   $\phi =$

4. Find a potential function, if possible, for the given vector fields.
   (a) $F(x, y, z) = (x, y, z)$. Find ϕ such that $\nabla\phi = F$.
   $\phi = \frac{x^2}{2} + \frac{y^2}{2} + \frac{z^2}{2}$
   (b) $F(x, y, z) = (2xy, x^2, 1)$. Find ϕ such that $\nabla\phi = F$.
   $\phi = x^2y + z$
   (c) $F(x, y, z) = (2xy + z + 1, 4z^2 + x^2, 6yz + x)$. Find ϕ such that $\nabla\phi = F$.
   $\phi = x + x^2y + xz + 3yz^2$
   (d) $F(x, y, z) = (y\cos xy, x\cos xy, 1)$. Find ϕ such that $\nabla\phi = F$.
   $\phi = z + \sin xy$
   (e) $F(x, y, z) = (1/(y + 2z), -x/(y + 2z)^2, -2z/(y + 2z)^2)$. Find ϕ such that $\nabla\phi = F$.
   $\phi = x/(y + 2z)$
   (f) $F(x, y, z) = (z + 2x, 1, x)$. Find ϕ such that $\nabla\phi = F$.
   $\phi = x^2 + y + xz$

5. Find a potential function, if possible, for the given vector fields.
   (a) $F(x, y, z) = (y, x, 0)$. Find ϕ such that $\nabla\phi = F$.
   $\phi =$
   (b) $F(x, y, z) = (y, -x, 0)$. Find ϕ such that $\nabla\phi = F$.
   $\phi =$
   (c) $F(x, y, z) = (x, y, y)$. Find ϕ such that $\nabla\phi = F$.
   $\phi =$
   (d) $F(x, y, z) = (x, x, yz)$. Find ϕ such that $\nabla\phi = F$.
   $\phi =$

5. Find a potential function, if possible, for the given vector fields.
   (a) $F(x, y, z) = (y, x, 0)$. Find ϕ such that $\nabla\phi = F$.
   $\phi = xy$
   (b) $F(x, y, z) = (y, -x, 0)$. Find ϕ such that $\nabla\phi = F$.
   $\phi = \text{DNE}$
   (c) $F(x, y, z) = (x, y, y)$. Find ϕ such that $\nabla\phi = F$.
   $\phi = \text{DNE}$
   (d) $F(x, y, z) = (x, x, yz)$. Find ϕ such that $\nabla\phi = F$.
   $\phi = \text{DNE}$