Math 233 - October 9, 2009

- How do you find a potential function?
- Does every vector field have a potential function?

1. How many zeros are at the end of the decimal representation for the number

\[ 100! = 100 \times 99 \times 98 \times 97 \times \cdots \times 2 \times 1 \]

Lecture Problems

2. Compute the curl and divergence of the vector fields

(a) \( F = (x, y, z) \), curl \( F = \)
\[ \text{div} \; F = \]
(b) \( F = (x^2, y^2, z^2) \), curl \( F = \)
\[ \text{div} \; F = \]
(c) \( F = (y, x, 1) \), curl \( F = \)
\[ \text{div} \; F = \]
(d) \( F = (-y, x, 1) \), curl \( F = \)
\[ \text{div} \; F = \]
(e) \( F = (-yz, xy, z) \), curl \( F = \)
\[ \text{div} \; F = \]

3. Find a potential function, if possible, for the given vector fields.

(a) \( F(x, y) = (y^2, 2xy - 1) \). Find \( \phi \) such that \( \nabla \phi = F \). \( \phi = \)
(b) \( F(x, y) = (2xy, x^2 - \sin y) \). Find \( \phi \) such that \( \nabla \phi = F \). \( \phi = \)
(c) \( F(x, y) = (1 + ye^x, e^x) \). Find \( \phi \) such that \( \nabla \phi = F \). \( \phi = \)
(d) \( F(x, y) = (1 + \sin(2y - x), -2\sin(2y - x)) \). Find \( \phi \) such that \( \nabla \phi = F \). \( \phi = \)

4. Find a potential function, if possible, for the given vector fields.

(a) \( F(x, y, z) = (x, y, z) \). Find \( \phi \) such that \( \nabla \phi = F \). \( \phi = \)
(b) \( F(x, y, z) = (2xy, x^2, 1) \). Find \( \phi \) such that \( \nabla \phi = F \). \( \phi = \)
(c) \( F(x, y, z) = (2xy + z + 1, 4z^2 + x^2, 6yz + x) \). Find \( \phi \) such that \( \nabla \phi = F \). \( \phi = \)
(d) \( F(x, y, z) = (y \cos xy, x \cos xy, 1) \). Find \( \phi \) such that \( \nabla \phi = F \). \( \phi = \)
(e) \( F(x, y, z) = (1/(y + 2z), -x/(y + 2z)^2, -2z/(y + 2z)^2) \). Find \( \phi \) such that \( \nabla \phi = F \). \( \phi = \)
(f) \( F(x, y, z) = (z + 2x, 1, x) \). Find \( \phi \) such that \( \nabla \phi = F \). \( \phi = \)

5. Find a potential function, if possible, for the given vector fields.

(a) \( F(x, y, z) = (y, x, 0) \). Find \( \phi \) such that \( \nabla \phi = F \). \( \phi = \)
(b) \( F(x, y, z) = (y, -x, 0) \). Find \( \phi \) such that \( \nabla \phi = F \). \( \phi = \)
(c) \( F(x, y, z) = (x, y, y) \). Find \( \phi \) such that \( \nabla \phi = F \). \( \phi = \)
(d) \( F(x, y, z) = (x, x, yz) \). Find \( \phi \) such that \( \nabla \phi = F \). \( \phi = \)