

Math 233 - October 8, 2009

Solutions

- What is the general procedure for finding extrema?

1. (a) What are the different types of extrema problems we have seen?

Solution: These types of problems are not mutually exclusive

- Max/min of a function over a closed and bounded set.
- Max/min of a function on an unbounded set (you are not sure you will have a global max/min).
- Lagrange multipliers
- Local max/mins–2nd derivative test.

- (b) How can you tell which type of extrema problem you are dealing with?

- (c) How do you deal differently with each type of problem?

- (d) What is the relation of max/mins to Taylor polynomials? (I.e., why did we spend all that time talking about Taylor polynomials?)

Lecture Problems

For each of the following extrema problems, determine what sort of problem it is, the domain in question, and what needs to be done (before you do it).

2. Find and classify the critical points of the function

$$f(x, y) = x^4 + y^4 - 4xy + 1$$

Solution: Note the domain is $\mathbb{R}^2$, an unbounded set.

P	f(P)	f _{xx} (P)	D(P)	Conclusion
(0, 0)	1	0	-16	Saddle
(1, 1)	-1	12	128	Local Min
(-1, -1)	-1	12	128	Local Min

3. A rectangular box, with a lid, is to be made from 12 square meters of cardboard. Find the maximum volume of such a box.

Solution: Need to maximize $f(x, y, z) = xyz$ with the constraint $2xz + 2yz + xy = 12$. Can use Lagrange multipliers to find the max occurs at $(2, 2, 1)$ with volume of 4 cubic meters.

4. Find the extrema of

$$f(x, y) = x^2 - 2xy + 2y$$

on the set

$$R = \{(x, y) | 0 \leq x \leq 3; 0 \leq y \leq 2\}$$

Solution: Max of 9 at $(3, 0)$. Min of 0 at $(0, 0)$ and $(2, 2)$.

5. Find and classify the critical points of the function

$$f(x, y) = x^3y + 12x^2 - 8y$$

Solution:

P	f(P)	f_{xx}(P)	D(P)	Conclusion
(2, -4)	48	-24	-144	Saddle

6. Find and classify the critical points of the function

$$f(x, y) = (2x - x^2)(2y - y^2)$$

Solution:

P	f(P)	f_{xx}(P)	D(P)	Conclusion
(0, 0)	0	0	-16	Saddle
(0, 2)	0	0	-16	Saddle
(2, 0)	0	0	-16	Saddle
(2, 2)	0	0	-16	Saddle
(1, 1)	1	-2	4	Local Max

7. Find three positive numbers whose sum is 100 and whose product is maximal.

Solution: We need to maximize $f(x, y, z) = xyz$ subject to $x + y + z = 100$. This is a good setup for Lagrange multipliers (do you see any other way to solve the problem?).

The maximum occurs when $x = y = z = \frac{100}{3}$.

8. Find and classify the critical points of the function

$$f(x, y) = xy(2x + 4y + 1)$$

Solution:

P	f(P)	f_{xx}(P)	D(P)	Conclusion
(0, 0)	0	0	-1	Saddle
(0, -1/4)	0	-1	-1	Saddle
(-1/2, 0)	0	0	-1	Saddle
(-1/6, -1/12)	1/216	-1/3	1/3	Local Max

9. A rectangular box with an open top is to be made to have volume of 256 cubic feet. Find the dimensions that require the least amount of material.

Solution: You are to minimize $f(x, y, z) = xy + 2yz + 2xz$ subject to $xyz = 256$. This is a good fit for Lagrange multipliers.

You should find that $(8, 8, 4)$ gives the minimum surface area.

10. Suppose the temperature on a circular plate

$$P = \{X \in \mathbb{R}^2 \mid \|X\| \leq \sqrt{2}\}$$

is given by $T(x, y) = x^2 + 2y^2 - 2x$. Find the maximum and minimum temperature on the plate.

Solution:

P	f(P)	Conclusion
(1, 0)	-1	Min
$(-2/\sqrt{2}, 0)$	$4/\sqrt{2} + 2$	
$(2/\sqrt{2}, 0)$	$-4/\sqrt{2} + 2$	
(-1, 1)	5	Max
(-1, -1)	5	Max
(1, 0)	-1	Min
(1, 0)	-1	Min

11. A pentagon is formed by placing an isocles triangle on top of a rectangle. If the perimeter of the pentagon is fixed at 5, find the dimensions of the rectangle and the height of the triangle that give the pentagon of maximal area.
12. Suppose temperature in space is given by

$$T = 3xy + z^3 - 3z$$

Find the maximum and minimum temperature on the sphere $x^2 + y^2 + z^2 - 2z = 0$.

Solution:

P	f(P)	Conclusion
(0, 0, 0)	0	
(0, 0, 2)	2	Max
$(1/\sqrt{2}, 1/\sqrt{2}, 1)$	-1/2	
$(-1/\sqrt{2}, -1/\sqrt{2}, 1)$	-1/2	
$(-1/\sqrt{2}, 1/\sqrt{2}, 1)$	-7/2	Min
$(1/\sqrt{2}, -1/\sqrt{2}, 1)$	-7/2	Min