

Math 233 - October 8, 2009

- What is the general procedure for finding extrema?
- 1. (a) What are the different types of extrema problems we have seen?
(b) How can you tell which type of extrema problem you are dealing with?
(c) How do you deal differently with each type of problem?
(d) What is the relation of max/mins to Taylor polynomials? (I.e., why did we spend all that time talking about Taylor polynomials?)

Lecture Problems

For each of the following extrema problems, determine what sort of problem it is, the domain in question, and what needs to be done (before you do it).

2. Find and classify the critical points of the function

$$f(x, y) = x^4 + y^4 - 4xy + 1$$

3. A rectangular box, with a lid, is to be made from 12 square meters of cardboard. Find the maximum volume of such a box.
4. Find the extrema of

$$f(x, y) = x^2 - 2xy + 2y$$

on the set

$$R = \{(x, y) | 0 \leq x \leq 3; 0 \leq y \leq 2\}$$

5. Find and classify the critical points of the function

$$f(x, y) = x^3y + 12x^2 - 8y$$

6. Find and classify the critical points of the function

$$f(x, y) = (2x - x^2)(2y - y^2)$$

7. Find three positive numbers whose sum is 100 and whose product is maximal.
8. Find and classify the critical points of the function

$$f(x, y) = xy(2x + 4y + 1)$$

9. A rectangular box with an open top is to be made to have volume of 256 cubic feet. Find the dimensions that require the least amount of material.

10. Suppose the temperature on a circular plate

$$P = \{X \in \mathbb{R}^2 \mid \|X\| \leq \sqrt{2}\}$$

is given by $T(x, y) = x^2 + 2y^2 - 2x$. Find the maximum and minimum temperature on the plate.

11. A pentagon is formed by placing an isosceles triangle on top of a rectangle. If the perimeter of the pentagon is fixed at 5, find the dimensions of the rectangle and the height of the triangle that give the pentagon of maximal area.
12. Suppose temperature in space is given by

$$T = 3xy + z^3 - 3z$$

Find the maximum and minimum temperature on the sphere $x^2 + y^2 + z^2 - 2z = 0$.