Math 233 - October 5, 2009

- How can we tell the difference between critical points? i.e., is it a max, min or saddle?
- Why does the second derivative test work in 1-variable?
- What can we learn from the second derivative test in 1-variable?

1. $f(x) = 3x^4 - 4x^3 + 1$.
   (a) Find all critical points of f. Use the second derivative test to determine if these are maxima or minima or neither.
   (b) For each critical point, find the second order Taylor polynomial at that point.
   (c) Determine the nature of the critical point of the Taylor polynomials.
2. $f(x) = x^4 - 8x^2$.
   (a) Find all critical points of f. Use the second derivative test to determine if these are maxima or minima or neither.
   (b) For each critical point, find the second order Taylor polynomial at that point.
   (c) Determine the nature of the critical point of the Taylor polynomials.
3. If $f(x) = a + bx^2$ (for $b \neq 0$).
   (a) What are the critical points of $f(x)$?
   (b) What values of b will make sure that you have a minimum at your critical point?
   (c) What values of b will make sure that you have a maximum at your critical point?

Lecture Problems

4. Let $P = (a, b)$ and suppose you are given a function $f(x, y)$. What is the general formula for the first order Taylor series for the function at the point P?
5. Find the first order Taylor series for the functions at the given points
   (a) $f(x, y) = y^3 + x^2y - 1$ at $(1, -2)$.
   (b) $f(x, y) = \ln(1 - x + y)$ at $(1, 3)$.
6. Find the second order Taylor series for the functions at the given points
   (a) $f(x, y) = y^3 + x^2y - 1$ at $(1, -2)$.
   (b) $f(x, y) = \ln(1 - x + y)$ at $(1, 3)$.