Math 233 Warm Up Problems

October 1, 2009

Solutions

1. Find the max and min of the function $f(x,y) = x^2 + y$ on the triangle

$$T = \{(x,y) | 0 \leq x \leq 3; -2 \leq y \leq 1; x + y \leq 1\}$$

   (a) Graph T.

   (b) Find all critical points in the interior of T.
   Solution: None

   (c) Find all max/mins on the side of the triangle: $x = 0$ and $-2 \leq y \leq 1$.
   Solution: Max: 1. Min: -2.

   (d) Find all max/mins on the bottom of the triangle: $y = -2$ and $0 \leq x \leq 3$.
   Solution: Max: 7. Min: -2.

   (e) Find all max/mins on the hypotenuse of the triangle: $y = 1 - x$ and $0 \leq x \leq 3$.
   Solution: Max: 7. Min: 3/4.

   (f) Find the absolute max and mins on the triangle.
   Solution: Max: 7. Min: -2.

Lecture Problems

2. Find the max and mins of $f(x,y,z) = x + 2y - z$ on the sphere $x^2 + y^2 + z^2 = 4$.

   (a) Set up 4 equations with 4 unknowns.

$$\begin{aligned} 1 &= 2\lambda x \\ 2 &= 2\lambda y \\ -1 &= 2\lambda z \\ x^2 + y^2 + z^2 &= 4 \end{aligned}$$

   (b) Solve the equations for x, y, z and λ.
   $\lambda = -\frac{\sqrt{6}}{4} \implies \left(-\frac{\sqrt{6}}{3}, -\frac{2\sqrt{6}}{3}, \frac{\sqrt{6}}{3}\right)$ $\lambda = \frac{\sqrt{6}}{4} \implies \left(\frac{\sqrt{6}}{3}, \frac{2\sqrt{6}}{3}, -\frac{\sqrt{6}}{3}\right)$

   (c) Find the max and min of the function on the sphere.

$$f\left(-\frac{\sqrt{6}}{3}, -\frac{2\sqrt{6}}{3}, \frac{\sqrt{6}}{3}\right) = -2\sqrt{6} \qquad \text{Min}$$

$$f\left(\frac{\sqrt{6}}{3}, \frac{2\sqrt{6}}{3}, -\frac{\sqrt{6}}{3}\right) = 2\sqrt{6} \qquad \text{Max}$$