Cylindrical Coordinates

1. Find the area of an “elementary polar region”:
\[ \{(x, y)| r_1 \leq r \leq r_2; \theta_1 \leq \theta \leq \theta_2\} \]
\[ A = \frac{(r_2 + r_1)}{2}(r_2 - r_1)(\theta_2 - \theta_1) = \tau \Delta r \Delta \theta \]

Lecture Problems

2. Find the cylindrical equation for the cartesian equation \( x^2 + y^2 = 4 \)
   Solution: \( r = 2 \)

3. Find the Cartesian equation for the cylindrical equation \( z = 4 \)
   Solution: \( z = 4 \)

4. Find the cylindrical equation for the Cartesian equation \( x - y = 0 \).
   Solution: \( \theta = \pi/4 \)

5. Find the cylindrical equation for the cone \( z^2 = x^2 + y^2 \).
   Solution: \( z = r \)

6. Find the Cartesian equation of the cylindrical equation \( r^2 + 4z^2 = 16 \).
   Solution: \( x^2 + y^2 + 4z^2 = 16 \)

7. Find the Cartesian equation of the cylindrical equation \( r^2 \cos 2\theta = z \).
   Hint: \( \cos 2\theta = \cos^2 \theta - \sin^2 \theta \).
   Solution: \( x^2 - y^2 = z \)

8. Find the volume of the solid bounded above by the sphere centered at the origin having radius 5 and bounded below by the plane \( z = 4 \).
   \[ V = \int_0^{2\pi} \int_0^3 \int_0^{\sqrt{25-r^2}} r \, dz \, dr \, d\theta = \frac{14\pi}{3} \]

9. Find the volume of the solid bounded above by the plane \( z = y + 4 \), below by the \( xy \)-plane and laterally by the right circular cylinder with radius 4 and whose axis is the \( z \)-axis.
   \[ V = \int_0^{2\pi} \int_0^4 \int_0^{r \sin \theta + 4} r \, dz \, dr \, d\theta = 64\pi \]

10. Find the volume of the solid inside \( x^2 + y^2 = 4 \), outside \( x^2 + y^2 = 1 \), below \( z = 12 - x^2 - y^2 \) and above \( z = 0 \).
    \[ V = \int_0^{2\pi} \int_1^2 \int_0^{12-r^2} r \, dz \, dr \, d\theta = \frac{57\pi}{2} \]