Math 233 - November 30, 2009

- Cylindrical Coordinates

1. Find the area of an "elementary polar region":

$$\{(x, y) | r_1 \leq r \leq r_2; \theta_1 \leq \theta \leq \theta_2\}$$

$$A =$$

1. Find the area of an "elementary polar region":

$$\{(x, y)|r_1 \leq r \leq r_2; \theta_1 \leq \theta \leq \theta_2\}$$

$$A = \frac{(r_2 + r_1)}{2}(r_2 - r_1)(\theta_2 - \theta_1) = \overline{r}\Delta r \Delta\theta$$

Lecture Problems

2. Find the cylindrical equation for the cartesian equation $x^2 + y^2 = 4$
3. Find the Cartesian equation for the cylindrical equation $z = 4$
4. Find the cylindrical equation for the Cartesian equation $x - y = 0$.
5. Find the cylindrical equation for the cone $z^2 = x^2 + y^2$.
6. Find the Cartesian equation of the cylindrical equation $r^2 + 4z^2 = 16$.
7. Find the Cartesian equation of the cylindrical equation $r^2 \cos 2\theta = z$.
   Hint: $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$.

Lecture Problems

2. Find the cylindrical equation for the cartesian equation $x^2 + y^2 = 4$
   Solution: $r = 2$
3. Find the Cartesian equation for the cylindrical equation $z = 4$
   Solution: $z = 4$
4. Find the cylindrical equation for the Cartesian equation $x - y = 0$.
   Solution: $\theta = \pi/4$
5. Find the cylindrical equation for the cone $z^2 = x^2 + y^2$.
   Solution: $z = r$
6. Find the Cartesian equation of the cylindrical equation $r^2 + 4z^2 = 16$.
   Solution: $x^2 + y^2 + 4z^2 = 16$.
7. Find the Cartesian equation of the cylindrical equation $r^2 \cos 2\theta = z$.
   Hint: $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$.
   Solution: $x^2 - y^2 = z$.

8. Find the volume of of the solid bounded above by the sphere centered at the origin having radius 5 and bounded below by the plane $z = 4$.

$$V =$$

9. Find the volume of the solid bounded above by the plane $z = y + 4$, below by the xy-plane and laterally by the right circular cylinder with radius 4 and whose axis is the z-axis.

$$V =$$

10. Find the volume of the solid inside $x^2 + y^2 = 4$, outside $x^2 + y^2 = 1$, below $z = 12 - x^2 - y^2$ and above $z = 0$.

$$V =$$

8. Find the volume of of the solid bounded above by the sphere centered at the origin having radius 5 and bounded below by the plane $z = 4$.

$$V = \int_0^{2\pi} \int_0^3 \int_4^{\sqrt{25-r^2}} r \, dz \, dr \, d\theta = \frac{14\pi}{3}$$

9. Find the volume of the solid bounded above by the plane $z = y + 4$, below by the xy-plane and laterally by the right circular cylinder with radius 4 and whose axis is the z-axis.

$$V = \int_0^{2\pi} \int_0^4 \int_0^{r\sin\theta+4} r \, dz \, dr \, d\theta = 64\pi$$

10. Find the volume of the solid inside $x^2 + y^2 = 4$, outside $x^2 + y^2 = 1$, below $z = 12 - x^2 - y^2$ and above $z = 0$.

$$V = \int_0^{2\pi} \int_1^2 \int_0^{12-r^2} r \, dz \, dr \, d\theta = \frac{57\pi}{2}$$