Math 233 - November 23, 2009

- Triple integrals

1. Use a triple integral to find the volume of the region. Make sure you sketch the region. R is the tetrahedron with vertices $(0, 0, 0)$, $(3, 2, 0)$, $(0, 3, 0)$, $(0, 0, 2)$.
2. Find the average value of $f(x, y, z) = x + 2y - z$ on the tetrahedron with vertices $(0, 0, 0)$, $(3, 2, 0)$, $(0, 3, 0)$, $(0, 0, 2)$.
3. Use a triple integral to find the volume of the region. Make sure you sketch the region. R is the region bounded by the cylinder $x^2 + y^2 - 2y = 0$ and the planes $x - y = 0$, $z = 1$ and $z = 3$.

1. Use a triple integral to find the volume of the region. Make sure you sketch the region. R is the tetrahedron with vertices $(0,0,0)$, $(3,2,0)$, $(0,3,0)$, $(0,0,2)$.
   Solution:
   $$V = \int_0^3 \int_{2x/3}^{3-x/3} \int_0^{(18-2x-6y)/9} \; dz\; dy\; dx = 3$$
2. Find the average value of $f(x,y,z) = x + 2y - z$ on the tetrahedron with vertices $(0,0,0)$, $(3,2,0)$, $(0,3,0)$, $(0,0,2)$.
   Solution:
   $$f_{ave} = \frac{1}{3} \int_0^3 \int_{2x/3}^{3-x/3} \int_0^{(18-2x-6y)/9} x + 2y - z \; dz\; dy\; dx = \frac{11}{4}$$
3. Use a triple integral to find the volume of the region. Make sure you sketch the region. R is the region bounded by the cylinder $x^2 + y^2 - 2y = 0$ and the planes $x - y = 0$, $z = 1$ and $z = 3$.
   Solution: $V = \frac{\pi}{2} - 1$

4. Use a triple integral to find the volume of the region. Make sure you sketch the region. R is solid region in the first octant bounded by $y = 2x^2$ and $y + 4z = 8$.
5. Find the average value of the function $f(x, y, z) = xy$ on the solid region in the first octant bounded by $y = 2x^2$ and $y + 4z = 8$.

4. Use a triple integral to find the volume of the region. Make sure you sketch the region. R is solid region in the first octant bounded by $y = 2x^2$ and $y + 4z = 8$.
Solution:

$$V = \int_0^2 \int_{2x^2}^8 \int_0^{2-y/4} dz\; dy\; dx = \frac{128}{15}$$

5. Find the average value of the function $f(x, y, z) = xy$ on the solid region in the first octant bounded by $y = 2x^2$ and $y + 4z = 8$.
Solution:

$$V = \frac{15}{128} \int_0^2 \int_{2x^2}^8 \int_0^{2-y/4} xy\; dz\; dy\; dx = \frac{5}{2}$$

6. Use a triple integral to find the volume of the region. Make sure you sketch the region. R is the region bounded by $x^2 = y$, $z^2 = y$ and $y = 1$.
7. Use a triple integral to find the volume of the region. Make sure you sketch the region. R is region boundec between $z = 9 - x^2 - y^2$ and $z = 3x^2 + 3y^2 - 16$.

6. Use a triple integral to find the volume of the region. Make sure you sketch the region. R is the region bounded by $x^2 = y$, $z^2 = y$ and $y = 1$.
   Solution:

$$V = 2\int_0^1 \int_{x^2}^1 \int_{-\sqrt{y}}^{\sqrt{y}} dz\ dy\ dx = 2$$

7. Use a triple integral to find the volume of the region. Make sure you sketch the region. R is region boundec between $z = 9 - x^2 - y^2$ and $z = 3x^2 + 3y^2 - 16$.
   Solution:

$$V = \int_{-5/2}^{5/2} \int_{-\sqrt{25/4-x^2}}^{\sqrt{25/4-x^2}} \int_{3x^2+3y^2-16}^{9-x^2-y^2} dz\ dy\ dx = \frac{625\pi}{8}$$

8. Change the integral to an integral in the order $dz\ dy\ dx$.

$$\int_0^1 \int_0^{\sqrt{1-y^2}} \int_0^{\sqrt{1-y^2-z^2}} f(x,y,z)\ dx\ dz\ dy =$$

9. Change the integral to an integral in the order $dz\ dy\ dx$.

$$\int_0^2 \int_0^{4-2y} \int_0^{4-2y-z} f(x,y,z)\ dx\ dz\ dy =$$

8. Change the integral to an integral in the order $dz\ dy\ dx$.

$$\int_0^1 \int_0^{\sqrt{1-y^2}} \int_0^{\sqrt{1-y^2-z^2}} f(x,y,z)\ dx\ dz\ dy = \int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} f(x,y,z)\ dz\ dy\ dx$$

9. Change the integral to an integral in the order $dz\ dy\ dx$.

$$\int_0^2 \int_0^{4-2y} \int_0^{4-2y-z} f(x,y,z)\ dx\ dz\ dy = \int_0^4 \int_0^{2-x/2} \int_0^{4-2y-x} f(x,y,z)\ dz\ dy\ dx$$

10. Change the integral to an integral in the order $dy\ dx\ dz$.

$$\int_0^2 \int_0^{9-x^2} \int_0^{2-x} f(x, y, z)\ dz\ dy\ dx =$$

11. Let R be the solid in the first octant cut off from the square cylinder with sides $x = 0$, $x = 1$, $z = 0$ and $z = 1$, cut by the plane $2x + y + 2z = 6$. Do this in two ways
    (a) Integrating $dz\ dy\ dx$.
    (b) Integrating $dy\ dx\ dz$.

10. Change the integral to an integral in the order $dy\ dx\ dz$.

$$\int_0^2 \int_0^{9-x^2} \int_0^{2-x} f(x,y,z)\ dz\ dy\ dx = \int_0^2 \int_0^{2-z} \int_0^{9-x^2} f(x,y,z)\ dy\ dx\ dz$$

11. Let R be the solid in the first octant cut off from the square cylinder with sides $x=0$, $x=1$, $z=0$ and $z=1$, cut by the plane $2x+y+2z=6$. Do this in two ways

    (a) Integrating $dz\ dy\ dx$.

    (b) Integrating $dy\ dx\ dz$.

Solution: $V=4$