Math 233 - November 23, 2009

- Triple integrals
1. Use a triple integral to find the volume of the region. Make sure you sketch the region. \( R \) is the tetrahedron with vertices \((0, 0, 0), (3, 2, 0), (0, 3, 0), (0, 0, 2)\).

2. Find the average value of \( f(x, y, z) = x + 2y - z \) on the tetrahedron with vertices \((0, 0, 0), (3, 2, 0), (0, 3, 0), (0, 0, 2)\).

3. Use a triple integral to find the volume of the region. Make sure you sketch the region. \( R \) is the region bounded by the cylinder \( x^2 + y^2 - 2y = 0 \) and the planes \( x - y = 0 \), \( z = 1 \) and \( z = 3 \).
1. Use a triple integral to find the volume of the region. Make sure you sketch the region. \( R \) is the tetrahedron with vertices \((0, 0, 0), (3, 2, 0), (0, 3, 0), (0, 0, 2)\).

Solution:

\[
V = \int_0^3 \int_{2x/3}^{3-x/3} \int_0^{(18-2x-6y)/9} dz \, dy \, dx = 3
\]

2. Find the average value of \( f(x, y, z) = x + 2y - z \) on the tetrahedron with vertices \((0, 0, 0), (3, 2, 0), (0, 3, 0), (0, 0, 2)\).

Solution:

\[
f_{\text{ave}} = \frac{1}{3} \int_0^3 \int_{2x/3}^{3-x/3} \int_0^{(18-2x-6y)/9} x + 2y - z \, dz \, dy \, dx = \frac{11}{4}
\]

3. Use a triple integral to find the volume of the region. Make sure you sketch the region. \( R \) is the region bounded by the cylinder \( x^2 + y^2 - 2y = 0 \) and the planes \( x - y = 0, z = 1 \) and \( z = 3 \).

Solution: \( V = \frac{\pi}{2} - 1 \)
4. Use a triple integral to find the volume of the region. Make sure you sketch the region. \( R \) is solid region in the first octant bounded by \( y = 2x^2 \) and \( y + 4z = 8 \).

5. Find the average value of the function \( f(x, y, z) = xy \) on the solid region in the first octant bounded by \( y = 2x^2 \) and \( y + 4z = 8 \).
4. Use a triple integral to find the volume of the region. Make sure you sketch the region. \( R \) is solid region in the first octant bounded by \( y = 2x^2 \) and \( y + 4z = 8 \).

Solution:

\[
V = \int_0^2 \int_{2x^2}^8 \int_0^{2-y/4} dz \, dy \, dx = \frac{128}{15}
\]

5. Find the average value of the function \( f(x, y, z) = xy \) on the solid region in the first octant bounded by \( y = 2x^2 \) and \( y + 4z = 8 \).

Solution:

\[
V = \frac{15}{128} \int_0^2 \int_{2x^2}^8 \int_0^{2-y/4} xy \, dz \, dy \, dx = \frac{5}{2}
\]
6. Use a triple integral to find the volume of the region. Make sure you sketch the region. \( R \) is the region bounded by \( x^2 = y, \ z^2 = y \) and \( y = 1 \).

7. Use a triple integral to find the volume of the region. Make sure you sketch the region. \( R \) is region bounded between \( z = 9 - x^2 - y^2 \) and \( z = 3x^2 + 3y^2 - 16 \).
6. Use a triple integral to find the volume of the region. Make sure you sketch the region. \( R \) is the region bounded by \( x^2 = y \), \( z^2 = y \) and \( y = 1 \).

**Solution:**

\[
V = 2 \int_0^1 \int_{x^2}^1 \int_{-\sqrt{y}}^{\sqrt{y}} \, dz \, dy \, dx = 2
\]

7. Use a triple integral to find the volume of the region. Make sure you sketch the region. \( R \) is region bounded between \( z = 9 - x^2 - y^2 \) and \( z = 3x^2 + 3y^2 - 16 \).

**Solution:**

\[
V = \int_{-5/2}^{5/2} \int_{-\sqrt{25/4-x^2}}^{\sqrt{25/4-x^2}} \int_{3x^2+3y^2-16}^{9-x^2-y^2} \, dz \, dy \, dx = \frac{625\pi}{8}
\]
8. Change the integral to an integral in the order $dz\ dy\ dx$.

$$\int_0^1 \int_0^{\sqrt{1-y^2}} \int_0^{\sqrt{1-y^2-z^2}} f(x, y, z) \, dx \, dz \, dy =$$

9. Change the integral to an integral in the order $dz\ dy\ dx$.

$$\int_0^2 \int_0^{4-2y} \int_0^{4-2y-z} f(x, y, z) \, dx \, dz \, dy =$$
8. Change the integral to an integral in the order $dz\ dy\ dx$.

$$\int_{0}^{1} \int_{0}^{\sqrt{1-y^{2}}} \int_{0}^{\sqrt{1-y^{2}-z^{2}}} f(x, y, z)\ dx\ dz\ dy =$$

$$\int_{0}^{1} \int_{0}^{\sqrt{1-x^{2}}} \int_{0}^{\sqrt{1-x^{2}-y^{2}}} f(x, y, z)\ dz\ dy\ dx$$

9. Change the integral to an integral in the order $dz\ dy\ dx$.

$$\int_{0}^{2} \int_{0}^{4-2y} \int_{0}^{4-2y-z} f(x, y, z)\ dx\ dz\ dy =$$

$$\int_{0}^{4} \int_{0}^{2-x/2} \int_{0}^{4-2y-x} f(x, y, z)\ dz\ dy\ dx$$
10. Change the integral to an integral in the order $dy \, dx \, dz$.

$$\int_0^2 \int_0^{9-x^2} \int_0^{2-x} f(x, y, z) \, dz \, dy \, dx =$$

11. Let $R$ be the solid in the first octant cut off from the square cylinder with sides $x = 0$, $x = 1$, $z = 0$ and $z = 1$, cut by the plane $2x + y + 2z = 6$. Do this in two ways

(a) Integrating $dz \, dy \, dx$.
(b) Integrating $dy \, dx \, dz$. 
10. Change the integral to an integral in the order $dy \, dx \, dz$.

$$
\int_0^2 \int_0^{9-x^2} \int_0^{2-x} f(x, y, z) \, dz \, dy \, dx = \\
\int_0^2 \int_0^{2-z} \int_0^{9-x^2} f(x, y, z) \, dy \, dx \, dz
$$

11. Let $R$ be the solid in the first octant cut off from the square cylinder with sides $x = 0$, $x = 1$, $z = 0$ and $z = 1$, cut by the plane $2x + y + 2z = 6$. Do this in two ways

(a) Integrating $dz \, dy \, dx$.
(b) Integrating $dy \, dx \, dz$.

**Solution:** $V = 4$