

Math 233 - November 23, 2009

- Triple integrals

1. Use a triple integral to find the volume of the region. Make sure you sketch the region. R is the tetrahedron with vertices $(0, 0, 0)$, $(3, 2, 0)$, $(0, 3, 0)$, $(0, 0, 2)$.
2. Find the average value of $f(x, y, z) = x + 2y - z$ on the tetrahedron with vertices $(0, 0, 0)$, $(3, 2, 0)$, $(0, 3, 0)$, $(0, 0, 2)$.
3. Use a triple integral to find the volume of the region. Make sure you sketch the region. R is the region bounded by the cylinder $x^2 + y^2 - 2y = 0$ and the planes $x - y = 0$, $z = 1$ and $z = 3$.
4. Use a triple integral to find the volume of the region. Make sure you sketch the region. R is solid region in the first octant bounded by $y = 2x^2$ and $y + 4z = 8$.
5. Find the average value of the function $f(x, y, z) = xy$ on the solid region in the first octant bounded by $y = 2x^2$ and $y + 4z = 8$.
6. Use a triple integral to find the volume of the region. Make sure you sketch the region. R is the region bounded by $x^2 = y$, $z^2 = y$ and $y = 1$.
7. Use a triple integral to find the volume of the region. Make sure you sketch the region. R is region bounded between $z = 9 - x^2 - y^2$ and $z = 3x^2 + 3y^2 - 16$.
8. Change the integral to an integral in the order $dz \, dy \, dx$.

$$\int_0^1 \int_0^{\sqrt{1-y^2}} \int_0^{\sqrt{1-y^2-z^2}} f(x, y, z) \, dx \, dz \, dy =$$

9. Change the integral to an integral in the order $dz \, dy \, dx$.

$$\int_0^2 \int_0^{4-2y} \int_0^{4-2y-z} f(x, y, z) \, dx \, dz \, dy =$$

10. Change the integral to an integral in the order $dy \, dx \, dz$.

$$\int_0^2 \int_0^{9-x^2} \int_0^{2-x} f(x, y, z) \, dz \, dy \, dx =$$

11. Let R be the solid in the first octant cut off from the square cylinder with sides $x = 0$, $x = 1$, $z = 0$ and $z = 1$, cut by the plane $2x + y + 2z = 6$. Do this in two ways
 - (a) Integrating $dz \, dy \, dx$.
 - (b) Integrating $dy \, dx \, dz$.