

Math 233 - November 20, 2009
Solutions

- Triple integrals

1. Compute

$$\int_0^1 \int_0^3 \int_1^4 x + yz \, dz \, dy \, dx = \frac{153}{4}$$

2. Compute

$$\int_0^1 \int_0^z \int_y^{z+1} 4xy - z \, dx \, dy \, dz = \frac{19}{40}$$

Lecture Problems

3. Integrate the function $f(x, y, z) = x + yz$ over the box $[0, 1] \times [0, 3] \times [1, 4]$

$$\int_0^1 \int_0^3 \int_1^4 x + yz \, dz \, dy \, dx = \frac{153}{4}$$

4. Compute

$$\int_0^2 \int_0^{4-2y} \int_0^{4-x-2y} x + y + z \, dz \, dx \, dy = \frac{10}{3}$$

5. Let R be the region in the first octant bounded by the surface $z = 9 - x^2 - y^2$ and the coordinate planes. Write down $\iiint_R f(x, y, z) \, dV$ as an iterated integral.

Challenge: write the integral down in all possible orders

$$\begin{aligned} \iiint_R f(x, y, z) \, dV &= \\ &= \int_0^3 \int_0^{\sqrt{9-x^2}} \int_0^{9-x^2-y^2} f(x, y, z) \, dz \, dy \, dx \\ &= \int_0^3 \int_0^{9-x^2} \int_0^{\sqrt{z-9+x^2}} f(x, y, z) \, dy \, dz \, dx \\ &= \int_0^9 \int_0^{\sqrt{9-z}} \int_0^{\sqrt{z-9+x^2}} f(x, y, z) \, dy \, dx \, dz \end{aligned}$$