Math 233 - November 20, 2009

▶ Triple integrals
1. Compute
\[ \int_0^1 \int_0^3 \int_1^4 x + yz
dz
dy
dx = \]

2. Compute
\[ \int_0^1 \int_0^z \int_y^{z+1} 4xy - z
dx
dy
dz = \]
1. Compute

\[ \int_0^1 \int_0^3 \int_1^4 x + yz \, dz \, dy \, dx = \frac{153}{4} \]

2. Compute

\[ \int_0^1 \int_0^z \int_y^{z+1} 4xy - z \, dx \, dy \, dz = \frac{19}{40} \]
Lecture Problems

3. Integrate the function $f(x, y, z) = x + yz$ over the box $\left[0, 1\right] \times \left[0, 3\right] \times \left[1, 4\right]$
Lecture Problems

3. Integrate the function $f(x, y, z) = x + yz$ over the box $[0, 1] \times [0, 3] \times [1, 4]$

\[
\int_0^1 \int_0^3 \int_1^4 x + yz \, dz \, dy \, dx = \frac{153}{4}
\]
4. Compute

\[
\int_{0}^{2} \int_{0}^{4-2y} \int_{0}^{4-x-2y} (x + y + z) \, dz \, dx \, dy =
\]
4. Compute

\[
\int_0^2 \int_0^{4-2y} \int_0^{4-x-2y} x + y + z \, dz \, dx \, dy = \frac{10}{3}
\]
5. Let $R$ be the region in the first octant bounded by the surface $z = 9 - x^2 - y^2$ and the coordinate planes. Write down $\iiint_R f(x, y, z) \, dV$ as an iterated integral. Challenge: write the integral down in all possible orders.
5. Let $R$ be the region in the first octant bounded by the surface $z = 9 - x^2 - y^2$ and the coordinate planes. Write down $\iiint_{R} f(x, y, z) \, dV$ as an iterated integral. Challenge: write the integral down in all possible orders

\[
\iiint_{R} f(x, y, z) \, dV =
\]

\[
= \int_{0}^{3} \int_{0}^{\sqrt{9-x^2}} \int_{0}^{9-x^2-y^2} f(x, y, z) \, dz \, dy \, dx
\]

\[
= \int_{0}^{3} \int_{0}^{\sqrt{9-x^2}} \int_{0}^{\sqrt{z-9+x^2}} f(x, y, z) \, dy \, dz \, dx
\]

\[
= \int_{0}^{9} \int_{0}^{\sqrt{9-z}} \int_{0}^{\sqrt{z-9+x^2}} f(x, y, z) \, dy \, dx \, dz
\]