

Math 233 - November 19, 2009

Solutions

- Green's Theorem

$$\begin{aligned}\oint_{\partial R} p \, dx + q \, dy &= \iint_R \frac{\partial q}{\partial x} - \frac{\partial p}{\partial y} \, dA \\ &= \iint_R \text{Rot} \, (p, q) \, dA \\ &= \int_{\partial R} F \cdot \mathbf{u} \, ds \\ \oint_{\partial R} F \cdot \mathbf{n} \, ds &= \iint_R \text{Div} \, F \, dA\end{aligned}$$

1. (Fun problem) Imagine that all the integers between 1 and 100 are written down. You then cross out two numbers (say x and y), and in their place you write the number $x + y + xy$. You repeat this process until eventually only a single number remains. What are the possible values for that remaining number?

Solution:

9425947759838359420851623124482936 ...
7495623127947025437683278893534169 ...
7759931622147650308786159180834691 ...
1623490003549599583369706302603263 ...
9999999999999999999999999999999

2. Let $F = (x, y^2 + x)$. $\text{Div} \, F = 1 + 2y$
3. Let $F = (xy^2, y^2/x^3)$. $\text{Div} \, F = y^2 + 2y^2/x^3$

Lecture Problems

4. Let $F = (x^2, x + y)$. Compute the flux of F through the circle of radius 2, centered at the origin. Do this in two ways, as a line integral and as a double integral. (At the very least, set up the integrals.)

$$\begin{aligned}\oint_C F \cdot \mathbf{n} \, ds &= \int_0^{2\pi} (x^2, x + y) \cdot (2 \cos t, 2 \sin t) \, dt \\ &= \int_0^{2\pi} 8 \cos^3 t + 4 \cos t \sin t + 4 \sin^2 t \, dt = 4\pi \\ \iint_D \text{Div} \, F \, dA &= \int_0^{2\pi} \int_0^2 (2x + 1)r \, dr \, d\theta = 4\pi\end{aligned}$$

5. Let $F = (x^2 - y^2, x^2 + y^2)$. Compute the flux of F through the rectangle with vertices $(0, 0)$, $(5, 0)$, $(5, 3)$ and $(0, 3)$.

$$\begin{aligned}\oint_C F \cdot n \, ds &= \iint_D \operatorname{Div} F \, dA \\ &= \int_0^{2\pi} \int_0^2 (2x + 1)r \, dr \, d\theta = 120\end{aligned}$$