Math 233 - November 19, 2009

Green’s Theorem

\[
\oint_{\partial R} p \, dx + q \, dy = \iint_{R} \frac{\partial q}{\partial x} - \frac{\partial p}{\partial y} \, dA = \iint_{R} \text{Rot} (p, q) \, dA = \int_{\partial R} F \cdot \mathbf{u} \, ds
\]

\[
\oint_{\partial R} F \cdot \mathbf{n} \, ds = \iint_{R} \text{Div} \, F \, dA
\]
1. (Fun problem) Imagine that all the integers between 1 and 100 are written down. You then cross out two numbers (say $x$ and $y$), and in their place you write the number $x + y + xy$. You repeat this process until eventually only a single number remains. What are the possible values for that remaining number?

2. Let $F = (x, y^2 + x)$. $\text{Div } F =$

3. Let $F = (xy^2, y^2/x^3)$. $\text{Div } F =$
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**Solution:**

$$9425947759838359420851623124482936 \ldots$$

$$7495623127947025437683278893534169 \ldots$$

$$7759931622147650308786159180834691 \ldots$$

$$1623490003549599583369706302603263 \ldots$$

$$9999999999999999999999999999999$$

2. Let $F = (x, y^2 + x)$. Div $F = 1 + 2y$

3. Let $F = (xy^2, y^2/x^3)$. Div $F = y^2 + 2y^2/x^3$
Lecture Problems

4. Let \( F = (x^2, x + y) \). Compute the flux of \( F \) through the circle of radius 2, centered at the origin. Do this in two ways, as a line integral and as a double integral. (At the very least, set up the integrals.)
**Lecture Problems**

4. Let $F = (x^2, x + y)$. Compute the flux of $F$ through the circle of radius 2, centered at the origin. Do this in two ways, as a line integral and as a double integral. (At the very least, set up the integrals.)

\[
\oint_C F \cdot n \, ds = \int_0^{2\pi} (x^2, x + y) \cdot (2 \cos t, 2 \sin t) \, dt
\]

\[
= \int_0^{2\pi} 8 \cos^3 t + 4 \cos t \sin t + 4 \sin^2 t \, dt = 4\pi
\]

\[
\iint_D \text{Div} \, F \, dA = \int_0^{2\pi} \int_0^2 (2x + 1) r \, dr \, d\theta = 4\pi
\]
5. Let $F = (x^2 - y^2, x^2 + y^2)$. Compute the flux of $F$ through the rectangle with vertices $(0, 0)$, $(5, 0)$, $(5, 3)$ and $(0, 3)$. 
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$$\oint_C F \cdot n \, ds = \iint_D \text{Div} \, F \, dA$$

$$= \int_0^{2\pi} \int_0^2 (2x + 1) r \, dr \, d\theta = 120$$