Math 233 - November 19, 2009

- Green's Theorem

$$\oint_{\partial R} p\ dx + q\ dy = \iint_R \frac{\partial q}{\partial x} - \frac{\partial p}{\partial y}\ dA$$

$$= \iint_R \mathrm{Rot}\,(p, q)\ dA$$

$$= \int_{\partial R} F \cdot \mathbf{u}\ ds$$

$$\oint_{\partial R} F \cdot \mathbf{n}\ ds = \iint_R \mathrm{Div}\ F\ dA$$

1. (Fun problem) Imagine that all the integers between 1 and 100 are written down. You then cross out two numbers (say x and y), and in their place you write the number $x + y + xy$. You repeat this process until eventually only a single number remains. What are the possible values for that remaining number?
2. Let $F = (x, y^2 + x)$. $\text{Div } F =$
3. Let $F = (xy^2, y^2/x^3)$. $\text{Div } F =$

1. (Fun problem) Imagine that all the integers between 1 and 100 are written down. You then cross out two numbers (say x and y), and in their place you write the number $x + y + xy$. You repeat this process until eventually only a single number remains. What are the possible values for that remaining number?
Solution:

$$
\begin{aligned}
&94259477598383594208516231244829\,36\ldots \\
&\qquad 74956231279470254376832788935341\,69\ldots \\
&\qquad 77599316221476503087861591808346\,91\ldots \\
&\qquad 16234900035495995833697063026032\,63\ldots \\
&\qquad 999999999999999999999999
\end{aligned}
$$

2. Let $F = (x, y^2 + x)$. $\mathrm{Div}\ F = 1 + 2y$
3. Let $F = (xy^2, y^2/x^3)$. $\mathrm{Div}\ F = y^2 + 2y^2/x^3$

Lecture Problems

4. Let $F = (x^2, x + y)$. Compute the flux of F through the circle of radius 2, centered at the origin. Do this in two ways, as a line integral and as a double integral. (At the very least, set up the integrals.)

Lecture Problems

4. Let $F = (x^2, x + y)$. Compute the flux of F through the circle of radius 2, centered at the origin. Do this in two ways, as a line integral and as a double integral. (At the very least, set up the integrals.)

$$\oint_C F \cdot n \, ds = \int_0^{2\pi} (x^2, x + y) \cdot (2\cos t, 2\sin t) \, dt$$

$$= \int_0^{2\pi} 8\cos^3 t + 4\cos t \sin t + 4\sin^2 t \, dt = 4\pi$$

$$\iint_D \mathrm{Div} \, F dA = \int_0^{2\pi} \int_0^2 (2x + 1) r \, dr \, d\theta = 4\pi$$

5. Let $F = (x^2 - y^2, x^2 + y^2)$. Compute the flux of F through the rectangle with vertices $(0,0)$, $(5,0)$, $(5,3)$ and $(0,3)$.

5. Let $F = (x^2 - y^2, x^2 + y^2)$. Compute the flux of F through the rectangle with vertices $(0,0)$, $(5,0)$, $(5,3)$ and $(0,3)$.

$$\begin{aligned}
\oint_C F \cdot n \, ds &= \iint_D \mathrm{Div}\ F dA \\
&= \int_0^{2\pi} \int_0^2 (2x+1) r \, dr \, d\theta = 120
\end{aligned}$$