# Math 233 - November 17, 2009
## Solutions

- Exam 3: 8.1-10.1

1. Find the work done by the vector field $F = (2e^y, xe^y, (z+1)e^z)$ from the point $(0,0,0)$ to $(1,1,1)$.

   **Solution:** Can not tell from give information.

2. Find the volume of the solid under the plane $z = x$ and above the region in the $xy$ plane bounded by the $y = x$ and $y = x^2$.

   **Solution:** $1/12$

3. Find the volume inside the sphere $x^2 + y^2 + z^2 = 25$ and outside the cylinder $x^2 + y^2 = 9$.

   **Solution:** $\pi 4^4/3$.

4. Find the volume of the solid bounded by $z = 40 - x^2 - y^2$ and $z = 8 + x^2 + y^2$.

   **Solution:** $R$ is the disc in the plane $x^2 + y^2 \leq 16$. $\iint_R (40 - x^2 - y^2) - (8 + x^2 + y^2)\, dA = 256\pi$

5. Convert the equation $x^2 + 2y^2 = 1$ to polar coordinates.

   **Solution:** $r^2 \cos^2\theta + 2r^2 \sin^2\theta = 1$ gives $r^2 = \frac{1}{1+\sin^2\theta}$ and so $r = \sqrt{\frac{1}{1+\sin^2\theta}}$.

6. Set up (but do not work out) an integral for the volume of the solid bounded by $z = x^2 + y^2$ and $z = 1 - y^2$. (See if you can write the integral in both rectangular and polar coordinates.)

   **Solution:** $R$ is the region bounded by the ellipse $x^2 + 2y^2 = 1$.

$$\begin{aligned}
&\iint_R (1 - y^2) - (x^2 + y^2)\, dA \\
&= \int_{-1}^{1} \int_{-\sqrt{(1-x^2)/2}}^{\sqrt{(1-x^2)/2}} 1 - x^2 - 2y^2\, dy\, dx \\
&= \int_0^{2\pi} \int_0^{1/\sqrt{1+\sin^2\theta}} (1 - r^2 - r^2\sin^2\theta) r\, dr\, d\theta
\end{aligned}$$

7. Compute $\int_0^1 \int_y^1 e^{x^2}\, dx\, dy$

   **Solution:** $e/2 - 1/2$

8. Compute $\int_{-1}^1 \int_0^{\sqrt{1-x^2}} x^2(x^2 + y^2)^2\, dy\, dz$

   **Solution:** $\pi/16$

9. Find the volume of the solid in the first octant bounded by the planes $y = 0$, $z = 0$, $x + y = 2$ and the cylinder $y^2 + z^2 = 1$.

   **Solution:** $(3\pi - 2)/6$.

10. Find the volume of the tetrahedron bounded by the coordinate planes and $3x + 8y + 4z = 24$.

    **Solution:** 24

11. Set up the integral $\iint_R f(x, y)\ dA$ where $R$ is the region between the curves $y = x$ and $x = y^2 - y$

    **Solution:** $\int_0^2 \int_{y^2-y}^{y} f(x, y)\ dA$

12. Find $\iint_D 4x + 2\ dA$ where $D$ is the region in the $xy$-plane bounded by the curves $y = x^2$ and $y = 2x$.

    **Solution:** 8

13. Find the volume of he solid in the first octant bounded by $z = 9 - y^2$, $x = 0$ and $x = 2$.

    **Solution:** 36

14. Find $\iint_D x^2 e^{xy}$ where $D$ is the triangle in the $xy$-plane plane with vertices $(0, 0)$, $(1, 0)$ and $(1, 1)$.

    **Solution:** $e/2 - 1$

15. Find the volume of the solid bounded by the paraboloid $z = x^2 + y^2 + 4$, the cylinder $x^2 + y^2 = 4$ and the plane $z = 0$.

    **Solution:** $24\pi$

16. Find $\iint_R xy\ dA$ where $R$ is the unit disc $x^2 + y^2 \leq 1$.

    **Solution:** 0

17. Find the volume under the plane $z = y$ and above the region in the $xy$-plane bounded by the curve $x = y^2 + 1$ and the lines $y = -x$, $y = 0$ and $y = 1$.

    **Solution:** 13/12

18. Find $\iint_R x^2 + y^2\ dA$ where $R$ is the region in the plane above the $x$-axis and inside the spiral $r = \theta$ where $0 \leq \theta \leq \pi$.

    **Solution:** $\pi^5/20$

19. Find $\int_0^2 \int_{y/2}^1 ye^{x^2}\ dx\ dy$

    **Solution:** $2e/3 - 2/3$

20. Find $\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{1}{\sqrt{x^2+y^2}}\ dy\ dx$

    **Solution:** 2

21. Find the volume of the pyramid above the $xy$-plane bounded by the planes $3x+z = 12$, $-3x+z = 12$, $3y + z = 12$, $-3y + z = 12$ and $z = 0$.

    **Solution:** 256

22. Find $\int_0^1 \int_{\sqrt{x}}^1 \cos y^3\ dy\ dx$

    **Solution:** $(\sin 1)/3$

23. Let $D$ be the region bounded by the circle $x^2 + y^2 = 2x$. Find $\iint_D (x^2 + y^2)y \, dA$

    **Solution:** 0

24. Find $\int_0^1 \int_0^{\sqrt{1-x^2}} x^2 + y^2 \, dy \, dx$

    **Solution:** $\pi/8$

25. Find $\int_C x^2 y\sqrt{z} \, dz$ where $C$ is the curve $C(t) = (t^3, t, t^2)$.

    **Solution:** $1/5$

26. Let $C$ the curve on the sides of the square, from $(0, 0)$ to $(1, 0)$ to $(1, 1)$ to $(0, 1)$ and back to $(0, 0)$. Find $\oint_C e^y \, dx + 2xe^y \, dy$.

    **Solution:** $e - 1$

27. Find $\int_C y^2 \, dx + x \, dy$ where $C$ is the boundary of the square bounded by the coordinate lines and the lines $x = 1$ and $y = 1$, oriented counter-clockwise.

    **Solution:** 0.

28. Find the work done by the vector field $F = (y, -x)$ counter-clockwise around the region $x^2 + y^2 \leq 1$, $y \geq 1$.

    **Solution:** $-\pi$