

Math 233 - November 16, 2009
Solutions

• Exam 3: 8.1-10.1

1. Curve integrals
2. Reverse Path
3. Curve integrals and potential functions
4. Dependence of path, special vector field
5. Double integrals (setting up, changing order or integration, etc)
6. Polar coordinates and double integrals with polar coordinates
7. Green's Theorem (applications)

1. Let R be the rectangle $[1, 2] \times [0, \pi]$. Find $\iint_R x \sin xy \, dA$.

Solution: 1

2. Find the volume under the surface $z = 6x$ and above triangle in the xy -plane with vertices $(0, 0)$, $(0, 4)$ and $(2, 4)$.

Solution: 16

3. Compute $\int_0^{\ln 2} 2xe^{y+x^2} \, dx \, dy$

Solution: 4

4. Change the order of integration $\int_0^{16} \int_0^{x^{1/4}} f(x, y) \, dy \, dx$.

Solution: $\int_0^2 \int_{y^4}^{16} f(x, y) \, dx \, dy$

5. Let C be the curve consisting of the line segment from $(-1, 0)$ to $(1, 0)$ and the upper half of the unit circle back to $(-1, 0)$, oriented counter-clockwise. Find $\int_C y^2 \cos x \, dx + (2xy + 2y \sin x) \, dy$

Solution: $4/3$

6. Set up the integral for the volume under the surface $z = f(x, y)$ and above the region in the xy -plane bounded by $y = x$ and $x = y^2 - y$.

Solution: $\int_0^2 \int_{y^2-y}^y f(x, y) \, dx \, dy$

7. Find the volume of the region bounded by the paraboloids $z = x^2 + y^2$ and $z = 8 - (x^2 + y^2)$.

Solution: 16π .

8. Find the volume of the solid above the xy -plane and the paraboloid $z = 4 - (x^2 + y^2)$.

Solution: 8π

9. $R = \{(x, y) | 0 \leq x \leq 1; x^2 \leq y \leq x\}$ Find $\iint_R xy \, dA$.

Solution: $1/24$

10. Find the volume under the surface $z = x^2y$ and above the triangle in the xy -plane with vertices $(0, 0)$, $(2, 0)$, $(2, 2)$.

Solution: $16/5$.

11. Convert the integral to polar $\int_{-3}^0 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} x^3xy^2 \, dy \, dx$

Solution: $\int_{\pi/2}^{3\pi/2} \int_0^3 r^4 \cos \theta \, dr \, d\theta$

12. Let D be the region outside the unit circle $r = 1$ and inside the cardioid $r = 1 - \sin \theta$. Set up a double integral for the area of the region.

Solution: $\int_{\pi}^{2\pi} \int_1^{1-\sin \theta} r \, dr \, d\theta$

13. $\int_0^4 \int_0^{y/2} f(x, y) \, dx \, dy = \int_a^b \int_c^d f(x, y) \, dy \, dx$. What are a, b, c, d ?

Solution: $0, 2, 2x, 4$.

14. Let R be the region inside the circle $x^2 + y^2 = 1$ and above the line $y = -x$. Set up the integral $\iint_R x^2 + y^2 \, dA$ in polar and in rectangular coordinates. Find the integral.

Solution: $\int_{-\pi/4}^{3\pi/4} \int_0^1 r^3 \, dr \, d\theta = \pi/4 = \int_{-1/\sqrt{2}}^{1/\sqrt{2}} \int_{-x}^{\sqrt{1-x^2}} x^2 + y^2 \, dy \, dx + \int_{1/\sqrt{2}}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} x^2 + y^2 \, dy \, dx$

15. Find $\iint_R x \, dA$ where R is the region bounded by $y = 3x - x^2$ and $y = x^2 - 3x$.

Solution: $27/2$

16. Find $\int_0^1 \int_{x^2}^1 x^3 \sin y^3 \, dy \, dx$

Solution: $(1 - \cos 1)/12$.

17. Compute $\iint_R x \, dA$ where R is the part of the disk $x^2 + y^2 \leq 9$, $x \geq 0$ and $y \geq x$.

Solution: $9(1 - 1/\sqrt{2})$

18. Find $\iint_R |r - 1| \, dA$ where R is the disk $r \leq 2$.

Solution: 2π

19. Find the volume of the solid with the plane $z = 0$ on the bottom, the cylinder $x^2 + y^2 = 4$ as the side and the plane $z = 3 - x - y$ as the top.

Solution: 12π

20. Find $\iint_R \sqrt{x^2 + y^2} \, dA$ where R is the region bounded by $xx^2 + y^2 = 2x$.

Solution: $32/9$

21. Find the line integral $\int_C x^3y^2 \, dz$ where C is the curve $x = 2t$, $y = t^2$, $z = t^2$, $0 \leq t \leq 1$.

Solution: $16/9$

22. Find the work done by the field $F = (x^2y^3, x^3y^2)$ from $(0, 0)$ to $(2, 1)$

Solution: $8/3$.

23. Find $\int_C \nabla f \cdot dr$ for some function f where $C(t) = t^2 + 1, t^3 + t$, $0 \leq t \leq 1$. You also know the following $f(0, 0) = 1$, $f(1, 1) = 5$, $f(2, 2) = 9$, $f(1, 0) = 3$, $f(1, 2) = 7$, $f(1, 3) = 4$.

Solution: 6

24. Find $\int_C F \cdot dr$ where $F = (x^2, xy, z^2)$ and C is the curve with $C(t) = (\sin t, \cos t, t^2)$, $0 \leq t \leq 2\pi$.

Solution: $64\pi^6/3$

25. Find $\int_C F \cdot dr$ where $F = (4xe^z, \cos y, 2x^2e^z)$ and C is the curve with $C(t) = (\sin \pi t, \pi t/2, t^2)$, $0 \leq t \leq 1$.

Solution: F is conservative. 1

26. Let R be the triangle with vertices $(0, 2)$, $(0, 1)$, and $(3, 2)$. Set up the integral $\iint_R f(x, y) dA = \int \int f(x, y) dy dx$.

Solution: $\int_0^3 \int_{x/3+1}^2 f(x, y) dy dx$

27. Find the work done by the vector field $F = (e^y, xe^y, (z+1)e^z)$ from the point $(0, 0, 0)$ to $(1, 1, 1)$.

Solution: $2e$