Math 233 - November 12, 2009

Solutions

- If $F = (p, q)$ then Rot $F = q_x - p_y$
- Green's Theorem
$$\oint_{\partial R} F = \iint_R \text{Rot } F \, dA$$
- Green's Theorem on regions with holes
- Other line integrals
- Other ways to write Green's Theorem

1. Let $F(x, y) = (x, xy^2)$. Find the rotation of F.
$$\text{Rot } F = y^2$$

2. Let $F(x, y) = (x^3 y, x/y)$. Find the rotation of F.
$$\text{Rot } F = \frac{1}{y} - x^3$$

3. Let
$$R = \{(x, y) | 1 \leq x^2 + y^2 \leq 2\}$$
   (a) Graph the region R.
   (b) What is the boundary of R?
   (c) Does it make sense to compute a curve integral along the boundary of R, ∂R?
   (d) What would "oriented positively" mean for the boundary of R?
   (e) Will Green's Theorem work on the region R? If so, how?

Lecture Problems

4. Let $r(t) = (t, t^3)$. At time t, find a vector that is tangent to the curve at $r(t)$? Can you find a unit tangent vector?
$$\text{Tangent Vector: } r'(t) = (1, 4t^2)$$
$$\text{Unit Tangent Vector: } u(t) = \frac{r'(t)}{\|r'(t)\|} = \frac{1}{\sqrt{1 + 16t^4}}(1, 4t^2)$$

5. Let $C(t) = (3t, t^3)$, $0 \leq t \leq 1$. Find
$$\int_C x^3 + y \, ds = \int_0^1 (9t^3 + t^3)\sqrt{9 + 9t^4} \, dt = 28\sqrt{2} - 14$$

6. Let C be the line segment from $(0, 0)$ to $(\pi, 2\pi)$. Find
$$\int_C \sin x + \cos y \, ds = \int_0^1 (\sin t + \cos(2t))\sqrt{5} \, dt = 2\sqrt{5}$$