Math 233 - November 12, 2009
Solutions

• If $F = (p, q)$ then $\text{Rot } F = q_x - p_y$

• Green’s Theorem
\[ \oint_{\partial R} F = \iint_R \text{Rot } F \, dA \]

• Green’s Theorem on regions with holes
• Other line integrals
• Other ways to write Green’s Theorem

1. Let $F(x, y) = (x, xy^2)$. Find the rotation of $F$.
\[ \text{Rot } F = y^2 \]

2. Let $F(x, y) = (x^3y, x/y)$. Find the rotation of $F$.
\[ \text{Rot } F = \frac{1}{y} - x^3 \]

3. Let
\[ R = \{(x, y)|1 \leq x^2 + y^2 \leq 2\} \]
   (a) Graph the region $R$.
   (b) What is the boundary of $R$?
   (c) Does it make sense to compute a curve integral along the boundary of $R$, $\partial R$?
   (d) What would “oriented positively” mean for the boundary of $R$?
   (e) Will Green’s Theorem work on the region $R$? If so, how?

Lecture Problems

4. Let $r(t) = (t, t^3)$. At time $t$, find a vector that is tangent to the curve at $r(t)$? Can you find a unit tangent vector?

   Tangent Vector: $r'(t) = (1, 4t^2)$
   Unit Tangent Vector: $u(t) = \frac{r'(t)}{\|r'(t)\|} = \frac{1}{\sqrt{1 + 16t^4}}(1, 4t^2)$

5. Let $C(t) = (3t, t^3)$, $0 \leq t \leq 1$. Find
\[ \int_C x^3 + y \, ds = \int_0^1 (9t^3 + t^3)\sqrt{9 + 9t^4} \, dt = 28\sqrt{2} - 14 \]

6. Let $C$ be the line segment from $(0, 0)$ to $(\pi, 2\pi)$. Find
\[ \int_C \sin x + \cos y \, ds = \int_0^1 (\sin t + \cos(2t))\sqrt{5} \, dt = 2\sqrt{5} \]