Math 233 - November 12, 2009

- If $F = (p, q)$ then $\mathrm{Rot}\ F = q_x - p_y$
- Green's Theorem

$$\oint_{\partial R} F = \iint_R \mathrm{Rot}\ F\ dA$$

- Green's Theorem on regions with holes
- Other line integrals
- Other ways to write Green's Theorem

1. Let $F(x, y) = (x, xy^2)$. Find the rotation of F.

   $$\mathrm{Rot}\ F =$$

2. Let $F(x, y) = (x^3 y, x/y)$. Find the rotation of F.

   $$\mathrm{Rot}\ F =$$

1. Let $F(x, y) = (x, xy^2)$. Find the rotation of F.

$$\mathrm{Rot}\ F = y^2$$

2. Let $F(x, y) = (x^3 y, x/y)$. Find the rotation of F.

$$\mathrm{Rot}\ F = \frac{1}{y} - x^3$$

3. Let

$$R = \{(x, y) | 1 \leq x^2 + y^2 \leq 2\}$$

(a) Graph the region R.
(b) What is the boundary of R?
(c) Does it make sense to compute a curve integral along the boundary of R, ∂R?
(d) What would "oriented positively" mean for the boundary of R?
(e) Will Green's Theorem work on the region R? If so, how?

Lecture Problems

4. Let $r(t) = (t, t^3)$. At time t, find a vector that is tangent to the curve at $r(t)$? Can you find a unit tangent vector?

   Tangent Vector:

   Unit Tangent Vector:

5. Let $C(t) = (3t, t^3)$, $0 \leq t \leq 1$. Find

$$\int_C x^3 + y \; ds =$$

6. Let C be the line segment from $(0, 0)$ to $(\pi, 2\pi)$. Find

$$\int_C \sin x + \cos y \; ds =$$

Lecture Problems

4. Let $r(t) = (t, t^3)$. At time t, find a vector that is tangent to the curve at $r(t)$? Can you find a unit tangent vector?

$$\text{Tangent Vector: } r'(t) = (1, 4t^2)$$

$$\text{Unit Tangent Vector: } u(t) = \frac{r'(t)}{\|r'(t)\|} = \frac{1}{\sqrt{1+16t^4}}(1, 4t^2)$$

5. Let $C(t) = (3t, t^3)$, $0 \leq t \leq 1$. Find

$$\int_C x^3 + y \, ds = \int_0^1 (9t^3 + t^3)\sqrt{9 + 9t^4} \, dt = 28\sqrt{2} - 14$$

6. Let C be the line segment from $(0, 0)$ to $(\pi, 2\pi)$. Find

$$\int_C \sin x + \cos y \, ds = \int_0^1 (\sin t + \cos(2t))\sqrt{5} \, dt = 2\sqrt{5}$$