Math 233 - November 10, 2009

- Parametrizing curves
- Proof of Green's Theorem
- General version of Green's Theorem

1. Parametrize the following curves
   (a) The curve that is the graph of $y = x^3$ between $x = 1$ and $x = 4$, parametrized from left to right.

   $$r(x) =$$

   (b) The curve that is the graph of $x = \sin y$ between $y = 0$ and $y = \pi$, parametrized from bottom to top.

   $$r(y) =$$

   (c) The curve that is the graph of the polar equation $r = 1 + \sin\theta$.

   $$r(\theta) =$$

1. Parametrize the following curves
    (a) The curve that is the graph of $y = x^3$ between $x = 1$ and $x = 4$, parametrized from left to right.

    $$r(x) = (x, x^3) \qquad 1 \leq t \leq 4$$

    (b) The curve that is the graph of $x = \sin y$ between $y = 0$ and $y = \pi$, parametrized from bottom to top.

    $$r(y) = (\sin y, y) \qquad 0 \leq t \leq \pi$$

    (c) The curve that is the graph of the polar equation $r = 1 + \sin \theta$.

    $$r(\theta) = ((1 + \sin \theta) \cos \theta, (1 + \sin \theta) \sin \theta) \qquad 0 \leq t \leq 2\pi$$

2. Find the area enclosed by the graph $r = 1 + \sin\theta$.
   Can you do this by setting up a line integral?

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Solution: Standard way:

$$A = \int_0^{2\pi} \int_0^{1+\sin\theta} r \; dr \; d\theta == \int_0^{2\pi} \frac{1}{2}(1+\sin\theta)^2 \; d\theta = \frac{3\pi}{2}$$

Line integral, use the parametrization from Problem 1:

$$\begin{aligned} A =& \frac{1}{2} \oint_r -y \; dx + x \; dy \\ =& \frac{1}{2} \int_0^{2\pi} -(1+\sin\theta)\sin\theta(\cos^2 t - sin^2 t - \sin t) \\ & \qquad\qquad + (1+\sin\theta)\cos\theta(2\cos t \sin t + \cos t) \; dt \\ =& \frac{3\pi}{2} \end{aligned}$$

Lecture Problems

3. Let C be the rectangle with vertices $(2, 1)$, $(6, 1)$, $(6, 4)$ and $(2, 4)$, oriented positively. Use Green's Theorem to find the line integral:

$$\oint_C (e^{3x} + 2y)\, dx + (x^2 + \sin y)\, dy =$$

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$$\oint_C (e^{3x} + 2y)\, dx + (x^2 + \sin y)\, dy = \int_2^6 \int_1^4 2x - 2\; dy\; dx = 72$$

4. Let C be the square with vertices $(0,0)$, $(1,0)$, $(1,1)$ and $(0,1)$. Find the work done by the vector field $F = (y, x^2y)$ around C in the counter-clockwise direction.

4. Let C be the square with vertices $(0,0)$, $(1,0)$, $(1,1)$ and $(0,1)$. Find the work done by the vector field $F = (y, x^2y)$ around C in the counter-clockwise direction.
Solution:

$$\oint_C F \cdot dr = \int_0^1 \int_0^1 2xy - 1 \; dA = -\frac{1}{2}$$