

Math 233 - November 9, 2009

Solutions

- Infinite double integral trick
- Proof of Green's Theorem

1. Let $F(x, y, z) = (p(x, y), q(x, y), 0)$ be a vector field in $\mathbb{R}^3$. Compute the curl of F .

$$\text{Curl } F = (0, 0, q_x - p_y)$$

2. Use Green's Theorem to compute. Let C be the boundary of the circle $x^2 + (y - 9)^2 = 4$ oriented clockwise.

$$\int_C \left(2y + \frac{x}{x^4 + 1} \right) dx + (\ln(1 + y^2) - x) dy = - \iint_R (-3) dA \\ = 12\pi$$

3. Compute

$$\int_{-\infty}^{\infty} e^{-x^2} dx =$$

Lecture Problems

4. Use a line integral to compute area of the ellipse

$$\frac{x^2}{25} + \frac{y^2}{49} = 1$$

(Parametrize the ellipse and then compute the appropriate line integral.)

Solution: $r(t) = (5 \cos t, 7 \sin t)$, $0 \leq t \leq 2\pi$.

$$-\frac{1}{2} \int_r x dy - y dx = 35\pi$$