Math 233 - November 9, 2009

- Infinite double integral trick
- Proof of Green’s Theorem
1. Let $F(x, y, z) = (p(x, y), q(x, y), 0)$ be a vector field in $\mathbb{R}^3$. Compute the curl of $F$.

   Curl $F =$

2. Use Green’s Theorem to compute. Let $C$ be the boundary of the circle $x^2 + (y - 9)^2 = 4$ oriented clockwise.

   $\int_C \left( 2y + \frac{x}{x^4 + 1} \right) \, dx + (\ln(1 + y^2) - x) \, dy =$

3. Compute

   $\int_{-\infty}^{\infty} e^{-x^2} \, dx =$
1. Let $F(x, y, z) = (p(x, y), q(x, y), 0)$ be a vector field in $\mathbb{R}^3$. Compute the curl of $F$.

$$\text{Curl } F = (0, 0, q_x - p_y)$$

2. Use Green’s Theorem to compute. Let $C$ be the boundary of the circle $x^2 + (y - 9)^2 = 4$ oriented clockwise.

$$\int_C \left( 2y + \frac{x}{x^4 + 1} \right) \, dx + (\ln(1 + y^2) - x) \, dy = - \iint_R (-3) \, dA = 12\pi$$

3. Compute

$$\int_{-\infty}^{\infty} e^{-x^2} \, dx =$$
Lecture Problems

4. Use a line integral to compute area of the ellipse

\[ \frac{x^2}{25} + \frac{y^2}{49} = 1 \]

(Parametrize the ellipse and then compute the appropriate line integral.)
Lecture Problems

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(Parametrize the ellipse and then compute the appropriate line integral.)

Solution: \( r(t) = (5 \cos t, 7 \sin t), \ 0 \leq t \leq 2\pi. \)

\[ -\frac{1}{2} \int_r x \ dy - y \ dx = 35\pi \]