Math 233 - November 9, 2009

- Infinite double integral trick
- Proof of Green's Theorem

1. Let $F(x, y, z) = (p(x, y), q(x, y), 0)$ be a vector field in $\mathbb{R}^3$. Compute the curl of F.

$$\mathrm{Curl}\ F =$$

2. Use Green's Theorem to compute. Let C be the boundary of the circle $x^2 + (y-9)^2 = 4$ oriented clockwise.

$$\int_C \left(2y + \frac{x}{x^4+1}\right)\, dx + (\ln(1+y^2) - x)\, dy =$$

3. Compute

$$\int_{-\infty}^{\infty} e^{-x^2}\, dx =$$

1. Let $F(x, y, z) = (p(x, y), q(x, y), 0)$ be a vector field in $\mathbb{R}^3$. Compute the curl of F.

$$\text{Curl } F = (0, 0, q_x - p_y)$$

2. Use Green's Theorem to compute. Let C be the boundary of the circle $x^2 + (y-9)^2 = 4$ oriented clockwise.

$$\int_C \left(2y + \frac{x}{x^4+1}\right) \, dx + (\ln(1+y^2) - x) \, dy = -\iint_R (-3) \, dA$$
$$= 12\pi$$

3. Compute

$$\int_{-\infty}^{\infty} e^{-x^2} \, dx =$$

Lecture Problems

4. Use a line integral to compute area of the ellipse

$$\frac{x^2}{25} + \frac{y^2}{49} = 1$$

(Parametize the ellipse and then compute the appropriate line integral.)

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Solution: $r(t) = (5\cos t, 7\sin t)$, $0 \leq t \leq 2\pi$.

$$-\frac{1}{2}\int_r x\ dy - y\ dx = 35\pi$$