Math 233 - November 6, 2009

- Polar integrals
- Green's Theorem!

1. Use rectangular coordinates to compute

$$\int_0^{\pi/4} \int_0^{\sec\theta} r^3 \sin^2\theta \, dr \, d\theta =$$

2. Use polar coordinates to compute the volume between the graphe of $z = 1 - x^2 - y^2$ and the xy-plane.

$$V =$$

3. Use polar coordinates to compute (this is probably not an integral that you would actually want to change to polar, but you should be able to anyway)

$$\int_1^2 \int_{-y}^{y} y \, dx \, dy =$$

1. Use rectangular coordinates to compute

$$\int_0^{\pi/4} \int_0^{\sec\theta} r^3 \sin^2\theta \, dr \, d\theta = \int_0^1 \int_0^x y^2 \, dy \, dx = \frac{1}{12}$$

2. Use polar coordinates to compute the volume between the graphe of $z = 1 - x^2 - y^2$ and the xy-plane.

$$V = \int_0^{2\pi} \int_0^1 (1 - r^2) r \, dr \, d\theta = \frac{\pi}{2}$$

3. Use polar coordinates to compute (this is probably not an integral that you would actually want to change to polar, but you should be able to anyway)

$$\int_1^2 \int_{-y}^{y} y \, dx \, dy = \int_{\pi/4}^{3\pi/4} \int_{\csc\theta}^{2\csc\theta} r^2 \sin\theta \, dr \, d\theta = \frac{14}{3}$$

Lecture Problems

4. Use Green's theorem to compute the line integrals

   (a) Let C be the closed curve oriented counter clockwise formed by $y = x/2$ and $y = \sqrt{2}$ between $(0,0)$ and $(4,2)$.

   $$\oint_C 2xy\ dx + y^2\ dy =$$

   (b) Let C be the closed curve oriented counter clockwise formed by $y = 0$ and $x = 2$ and $y = x^3/4$.

   $$\oint_C (2x + y^2)\ dx + (x^2 + 2y)\ dy =$$

   (c) Let C be the ellipse, $9x^2 + 16y^2 = 144$ oriented counter clockwise.

   $$\oint_C (x^2 + 4xy)\ dx + (2x^2 + 3y)\ dy =$$

Lecture Problems

4. Use Green's theorem to compute the line integrals
   (a) Let C be the closed curve oriented counter clockwise formed by $y = x/2$ and $y = \sqrt{2}$ between $(0,0)$ and $(4,2)$.

$$\oint_C 2xy\ dx + y^2\ dy = \int_0^2 \int_{y^2}^{2y} -2x\ dx\ dy = -\frac{64}{15}$$

   (b) Let C be the closed curve oriented counter clockwise formed by $y = 0$ and $x = 2$ and $y = x^3/4$.

$$\oint_C (2x + y^2)\ dx + (x^2 + 2y)\ dy = \int_0^2 \int_0^{x^3/4} 2x - 2y\ dy\ dx = \frac{72}{35}$$

   (c) Let C be the ellipse, $9x^2 + 16y^2 = 144$ oriented counter clockwise.

$$\oint_C (x^2 + 4xy)\ dx + (2x^2 + 3y)\ dy = \iint_R 0\ dA = 0$$