Math 233 - November 6, 2009

- Polar integrals
- Green’s Theorem!
1. Use rectangular coordinates to compute
\[ \int_0^{\pi/4} \int_0^{\sec \theta} r^3 \sin^2 \theta \, dr \, d\theta = \]

2. Use polar coordinates to compute the volume between the graph of \( z = 1 - x^2 - y^2 \) and the \( xy \)-plane.
\[ V = \]

3. Use polar coordinates to compute (this is probably not an integral that you would actually want to change to polar, but you should be able to anyway)
\[ \int_1^2 \int_{-y}^y y \, dx \, dy = \]
1. Use rectangular coordinates to compute

\[
\int_0^{\pi/4} \int_0^{\sec \theta} r^3 \sin^2 \theta \, dr \, d\theta = \int_0^1 \int_0^x y^2 \, dy \, dx = \frac{1}{12}
\]

2. Use polar coordinates to compute the volume between the graph of \( z = 1 - x^2 - y^2 \) and the xy-plane.

\[
V = \int_0^{2\pi} \int_0^1 (1 - r^2) r \, dr \, d\theta = \frac{\pi}{2}
\]

3. Use polar coordinates to compute (this is probably not an integral that you would actually want to change to polar, but you should be able to anyway)

\[
\int_1^2 \int_{-y}^y y \, dx \, dy = \int_{\pi/4}^{3\pi/4} \int_{\csc \theta}^2 r^2 \sin \theta \, dr \, d\theta = \frac{14}{3}
\]
Lecture Problems

4. Use Green’s theorem to compute the line integrals

(a) Let \( C \) be the closed curve oriented counter clockwise formed by \( y = x/2 \) and \( y = \sqrt{2} \) between \((0, 0)\) and \((4, 2)\).

\[
\oint_C 2xy \, dx + y^2 \, dy =
\]

(b) Let \( C \) be the closed curve oriented counter clockwise formed by \( y = 0 \) and \( x = 2 \) and \( y = x^3/4 \).

\[
\oint_C (2x + y^2) \, dx + (x^2 + 2y) \, dy =
\]

(c) Let \( C \) be the ellipse, \( 9x^2 + 16y^2 = 144 \) oriented counter clockwise.

\[
\oint_C (x^2 + 4xy) \, dx + (2x^2 + 3y) \, dy =
\]
Lecture Problems

4. Use Green’s theorem to compute the line integrals

(a) Let $C$ be the closed curve oriented counter clockwise formed by $y = x/2$ and $y = \sqrt{2}$ between $(0, 0)$ and $(4, 2)$.

$$\oint_C 2xy \, dx + y^2 \, dy = \int_0^2 \int_{y^2}^{2y} -2x \, dx \, dy = -\frac{64}{15}$$

(b) Let $C$ be the closed curve oriented counter clockwise formed by $y = 0$ and $x = 2$ and $y = x^3/4$.

$$\oint_C (2x + y^2) \, dx + (x^2 + 2y) \, dy = \int_0^2 \int_0^{x^3/4} 2x - 2y \, dy \, dx = \frac{72}{35}$$

(c) Let $C$ be the ellipse, $9x^2 + 16y^2 = 144$ oriented counter clockwise.

$$\oint_C (x^2 + 4xy) \, dx + (2x^2 + 3y) \, dy = \iint_R 0 \, dA = 0$$