Math 233 - November 5, 2009
Solutions

• Polar Day (Part 2)

1. An apple pie has radius 3 units. You take a slice of pie that is quarter of the whole pie, $\Delta \theta = \pi/2$. What is the area of your slice of pie?

   Solution: $\frac{9\pi}{4}$

2. Same pie. Now your slice has $\Delta \theta = \pi/4$. What is the area of your slice of pie?

   Solution: $\frac{9\pi}{8}$

3. General pie with radius $r$ and a slice with angle $\Delta \theta$. What is the area of your slice of pie?

   Solution: $\frac{\Delta \theta}{2} r^2$

Lecture Problems

4. Transform the following Cartesian equations into polar equations. (If possible, express $r$ as a function of $\theta$.)

   (a) $x = 1$

   Solution: $r = \sec \theta$

   (b) $x = 2$

   Solution: $r = 2 \sec \theta$

   (c) $y = 0$

   Solution: $\theta = 0$

   (d) $y = 1$

   Solution: $r = \csc \theta$

   (e) $x^2 + y^2 = 16$

   Solution: $r = 4$

   (f) $x^2 + (y - 1/2)^2 = 1/4$

   Solution: $r = \sin \theta$

   (g) $(x - 1/2)^2 + y^2 = 1/4$

   Solution: $r = \cos \theta$

5. Use polar coordinates to compute

\[
\int_0^1 \int_0^{\sqrt{1-x^2}} \frac{1}{\sqrt{4-x^2-y^2}} \, dy \, dx = \int_0^{\pi/2} \int_0^1 \frac{1}{\sqrt{4-r^2}} r \, dr \, d\theta
\]

\[
= \frac{(2 - \sqrt{3})\pi}{2}
\]
6. Use polar coordinates to compute

\[
\int_{1}^{2} \int_{0}^{\sqrt{2x-x^2}} \frac{1}{\sqrt{x^2 + y^2}} \, dy \, dx = \int_{0}^{\pi/4} \int_{\sec \theta}^{2 \cos \theta} \frac{1}{r} \, r \, dr \, d\theta \\
= \int_{0}^{\pi/4} 2 \cos \theta - \sec \theta \, dr \, d\theta = \frac{1}{\sqrt{2}} - \ln(1 + \sqrt{2})
\]

7. Use rectangular coordinates to compute

\[
\int_{0}^{\pi/4} \int_{0}^{\sec \theta} r^3 \sin^2 \theta \, dr \, d\theta =
\]