Math 233 - November 5, 2009

- Polar Day (Part 2)

1. An apple pie has radius 3 units. You take a slice of pie that is quarter of the whole pie, $\Delta\theta = \pi/2$. What is the area of your slice of pie?
2. Same pie. Now your slice has $\Delta\theta = \pi/4$. What is the area of your slice of pie?
3. General pie with radius r and a slice with angle $\Delta\theta$. What is the area of your slice of pie?

1. An apple pie has radius 3 units. You take a slice of pie that is quarter of the whole pie, $\Delta\theta = \pi/2$. What is the area of your slice of pie?
   Solution: $\frac{9\pi}{4}$
2. Same pie. Now your slice has $\Delta\theta = \pi/4$. What is the area of your slice of pie?
   Solution: $\frac{9\pi}{8}$
3. General pie with radius r and a slice with angle $\Delta\theta$. What is the area of your slice of pie?
   Solution: $\frac{\Delta\theta}{2} r^2$

Lecture Problems

4. Transform the following Cartesian equations into polar equations. (If possible, express r as a function of θ.)

   (a) $x = 1$
   (b) $x = 2$
   (c) $y = 0$
   (d) $y = 1$
   (e) $x^2 + y^2 = 16$
   (f) $x^2 + (y - 1/2)^2 = 1/4$
   (g) $(x - 1/2)^2 + y^2 = 1/4$

Lecture Problems

4. Transform the following Cartesian equations into polar equations. (If possible, express r as a function of θ.)

   (a) $x = 1$
   Solution: $r = \sec\theta$

   (b) $x = 2$
   Solution: $r = 2\sec\theta$

   (c) $y = 0$
   Solution: $\theta = 0$

   (d) $y = 1$
   Solution: $r = \csc\theta$

   (e) $x^2 + y^2 = 16$
   Solution: $r = 4$

   (f) $x^2 + (y - 1/2)^2 = 1/4$
   Solution: $r = \sin\theta$

   (g) $(x - 1/2)^2 + y^2 = 1/4$
   Solution: $r = \cos\theta$

5. Use polar coordinates to compute

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \frac{1}{\sqrt{4-x^2-y^2}}\,dy\,dx =$$

6. Use polar coordinates to compute

$$\int_1^2 \int_0^{\sqrt{2x-x^2}} \frac{1}{\sqrt{x^2+y^2}}\,dy\,dx =$$

7. Use rectangular coordinates to compute

$$\int_0^{\pi/4} \int_0^{\sec\theta} r^3 \sin^2\theta\,dr\,d\theta =$$

5. Use polar coordinates to compute

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \frac{1}{\sqrt{4-x^2-y^2}}\, dy\, dx = \int_0^{\pi/2} \int_0^1 \frac{1}{\sqrt{4-r^2}} r\, dr\, d\theta$$
$$= \frac{(2-\sqrt{3})\pi}{2}$$

6. Use polar coordinates to compute

$$\int_1^2 \int_0^{\sqrt{2x-x^2}} \frac{1}{\sqrt{x^2+y^2}}\, dy\, dx = \int_0^{\pi/4} \int_{\sec\theta}^{2\cos\theta} \frac{1}{r}\, r\, dr\, d\theta$$
$$= \int_0^{\pi/4} 2\cos\theta - \sec\theta\, dr\, d\theta = \frac{1}{\sqrt{2}} - \ln(1+\sqrt{2})$$

7. Use rectangular coordinates to compute

$$\int_0^{\pi/4} \int_0^{\sec\theta} r^3 \sin^2\theta\, dr\, d\theta =$$