Math 233 - November 3, 2009

Polar Day
1. Find the volume of the solid bounded by the cylinders \( x^2 + z^2 = 16, \ y^2 + z^2 = 16 \) and the coordinate planes.

2. Find the volume of the solid bounded by the cylinders \( x^2 + z^2 = 16, \ y^2 + z^2 = 16 \).
1. Find the volume of the solid bounded by the cylinders 
\( x^2 + z^2 = 16, \ y^2 + z^2 = 16 \) and the coordinate planes.

**Solution:**

\[
\int_{0}^{4} \int_{0}^{\sqrt{16-x^2}} dy \ dx + \int_{0}^{4} \int_{0}^{\sqrt{16-y^2}} dx \ dy = \frac{64}{3} + \frac{64}{3} = \frac{128}{3}
\]

Or, another method is

\[
\int_{0}^{4} \int_{0}^{\sqrt{16-z^2}} \sqrt{16-z^2} \ dy \ dz = \frac{128}{3}
\]

2. Find the volume of the solid bounded by the cylinders 
\( x^2 + z^2 = 16, \ y^2 + z^2 = 16 \).

**Solution:** This answer should be 8 times the previous answer, \( \frac{1024}{3} \).
Lecture Problems

3. Use the “Polar Map”, \( P : \mathbb{R}^2 \to \mathbb{R}^2 \) where
\[
P(\theta, r) = (r \cos \theta, r \sin \theta).
\]

(a) What is the image of the rectangle \([0, \pi] \times [0, 1]\).
(b) What is the image of the rectangle \([\pi/2, \pi] \times [1, 2]\).
(c) What is the image of the rectangle \([2\pi, 3\pi] \times [1, 5]\).
(d) What is the image of the rectangle \([0, \pi] \times [-1, 0]\).
(e) What is the image of the rectangle \([0, \pi] \times [-1, 1]\).
(f) What is the image of the line \( r = 1 \)?
(g) What is the image of the line \( r = -1 \)?
(h) What is the image of the line \( \theta = 1 \)?
(i) What is the image of the line \( \theta = -1 \)?
4. Transform the following Cartesian equations into polar equations. (If possible, express \( r \) as a function of \( \theta \).)

(a) \( x = 1 \)
(b) \( x = 2 \)
(c) \( y = 0 \)
(d) \( y = 1 \)
(e) \( x^2 + y^2 = 16 \)
(f) \( x^2 + (y - 1/2)^2 = 1/4 \)
(g) \( (x - 1/2)^2 + y^2 = 1/4 \)
4. Transform the following Cartesian equations into polar equations. (If possible, express $r$ as a function of $\theta$.)

(a) $x = 1$
Solution: $r = \sec \theta$

(b) $x = 2$
Solution: $r = 2 \sec \theta$

(c) $y = 0$
Solution: $\theta = 0$

(d) $y = 1$
Solution: $r = \csc \theta$

(e) $x^2 + y^2 = 16$
Solution: $r = 4$

(f) $x^2 + (y - 1/2)^2 = 1/4$
Solution: $r = \sin \theta$

(g) $(x - 1/2)^2 + y^2 = 1/4$
Solution: $r = \cos \theta$
5. Let $R$ be the unit disc.

(a) \[ \iint_R \, dA = \]

(b) \[ \int_0^{2\pi} \int_0^1 \, dr \, d\theta = \]
5. Let $R$ be the unit disc.

(a) \[ \int \int_{R} dA = \pi \]

(b) \[ \int_{0}^{2\pi} \int_{0}^{1} dr \, d\theta = 2\pi \]