Math 233 - November 3, 2009

- Polar Day

1. Find the volume of the solid bounded by the cylinders $x^2 + z^2 = 16$, $y^2 + z^2 = 16$ and the coordinate planes.
2. Find the volume of the solid bounded by the cylinders $x^2 + z^2 = 16$, $y^2 + z^2 = 16$.

1. Find the volume of the solid bounded by the cylinders $x^2 + z^2 = 16$, $y^2 + z^2 = 16$ and the coordinate planes.
   Solution:

$$\int_0^4 \int_0^x \sqrt{16 - x^2}\, dy\, dx + \int_0^4 \int_0^y \sqrt{16 - y^2}\, dx\, dy = \frac{64}{3} + \frac{64}{3} = \frac{128}{3}$$

   Or, another method is

$$\int_0^4 \int_0^{\sqrt{16-z^2}} \sqrt{16 - z^2}\, dy\, dz = \frac{128}{3}$$

2. Find the volume of the solid bounded by the cylinders $x^2 + z^2 = 16$, $y^2 + z^2 = 16$.
   Solution: This answer should be 8 times the previous answer, $\frac{1024}{3}$.

Lecture Problems

3. Use the "Polar Map", $P : \mathbb{R}^2 \to \mathbb{R}^2$ where $P(\theta, r) = (r\cos\theta, r\sin\theta)$.
   (a) What is the image of the rectangle $[0, \pi] \times [0, 1]$.
   (b) What is the image of the rectangle $[\pi/2, \pi] \times [1, 2]$.
   (c) What is the image of the rectangle $[2\pi, 3\pi] \times [1, 5]$.
   (d) What is the image of the rectangle $[0, \pi] \times [-1, 0]$.
   (e) What is the image of the rectangle $[0, \pi] \times [-1, 1]$.
   (f) What is the image of the line $r = 1$?
   (g) What is the image of the line $r = -1$?
   (h) What is the image of the line $\theta = 1$?
   (i) What is the image of the line $\theta = -1$?

4. Transform the following Cartesian equations into polar equations. (If possible, express r as a function of θ.)

   (a) $x = 1$
   (b) $x = 2$
   (c) $y = 0$
   (d) $y = 1$
   (e) $x^2 + y^2 = 16$
   (f) $x^2 + (y - 1/2)^2 = 1/4$
   (g) $(x - 1/2)^2 + y^2 = 1/4$

4. Transform the following Cartesian equations into polar equations. (If possible, express r as a function of θ.)

   (a) $x = 1$
   Solution: $r = \sec\theta$

   (b) $x = 2$
   Solution: $r = 2\sec\theta$

   (c) $y = 0$
   Solution: $\theta = 0$

   (d) $y = 1$
   Solution: $r = \csc\theta$

   (e) $x^2 + y^2 = 16$
   Solution: $r = 4$

   (f) $x^2 + (y - 1/2)^2 = 1/4$
   Solution: $r = \sin\theta$

   (g) $(x - 1/2)^2 + y^2 = 1/4$
   Solution: $r = \cos\theta$

5. Let R be the unit disc.

(a)

$$\iint_R dA =$$

(b)

$$\int_0^{2\pi} \int_0^1 dr\ d\theta =$$

5. Let R be the unit disc.

(a)

$$\iint_R dA = \pi$$

(b)

$$\int_0^{2\pi} \int_0^1 dr\ d\theta = 2\pi$$