Math 233 - November 2, 2009
Solutions

- Volume Day

1. Let R be the larger region (with negative x-values) enclosed by the curves $y = -x + 2$, $y = x^2$ and $y = 1/2$. Set up the integral $\iint_R f(x, y)\, dA$ in both ways.

$$\begin{aligned}\iint_R f(x,y)\,dA &= \int_{-2}^{-1/\sqrt{2}} \int_{x^2}^{-x+2} f(x,y)\,dy\,dx \\ &\quad + \int_{-1/\sqrt{2}}^{1/\sqrt{2}} \int_{1/2}^{-x+2} f(x,y)\,dy\,dx \\ &\quad + \int_{1/\sqrt{2}}^{1} \int_{x^2}^{-x+2} f(x,y)\,dy\,dx \\ &= \int_{1/2}^{1} \int_{-\sqrt{y}}^{\sqrt{y}} f(x,y)\,dx\,dy \\ &\quad + \int_{1}^{4} \int_{-\sqrt{y}}^{-y+2} f(x,y)\,dx\,dy\end{aligned}$$

Lecture Problems

2. Find the volume of the tetrahedron bounded by the coordinate planes and $8x + 2y + 3z = 24$.

   Solution: 48

3. Find the volume of the tetrahedron bounded by the coordinate planes and $-x + 2y - 3z = 24$.

   Solution: Remember to graph the tetrahedron! 384

4. Find the volume of the solid in the first octant bounded by the coordinate planes and the planes $2x + y - 4 = 0$ and $8x + y - 4z = 0$.

   Solution:

$$\int_0^3 \int_0^{-2x+4} \frac{1}{4}(8x + y)\,dy\,dx = \frac{20}{3}$$

5. Find the volume of the solid in the first octant bounded by the cylinder $y = x^2$ and the planes $x = 0$, $z = 0$ and $y + z = 1$.

   Solution:

$$\int_0^1 \int_{x^2}^1 1 - y\,dy\,dx = \frac{4}{15}$$

6. Find the volume of the solid bounded by $z = e^{x-y}$, the plane $x + y = 1$ and the coordinate planes.

   Solution:

$$\int_0^1 \int_0^{1-x} e^{x-y}\, dy\, dx = \frac{e}{2} + \frac{1}{2e} - 1$$

7. Find the volume of the solid bounded by the cylinders $x^2 + z^2 = 16$, $y^2 + z^2 = 16$ and the coordinate planes.