• Volume Day

1. Let \( R \) be the larger region (with negative \( x \)-values) enclosed by the curves \( y = -x + 2 \), \( y = x^2 \) and \( y = 1/2 \). Set up the integral \( \iint_R f(x, y) \, dA \) in both ways.

\[
\iint_R f(x, y) \, dA = \int_{-1/\sqrt{2}}^{1/\sqrt{2}} \int_{x^2}^{x+2} f(x, y) \, dy \, dx \\
+ \int_{-1/\sqrt{2}}^{1/\sqrt{2}} \int_{1/2}^{x+2} f(x, y) \, dy \, dx \\
+ \int_{1/\sqrt{2}}^{1} \int_{x^2}^{x+2} f(x, y) \, dy \, dx \\
= \int_{1/2}^{1} \int_{-\sqrt{y}}^{\sqrt{y}} f(x, y) \, dx \, dy \\
+ \int_{1}^{4} \int_{-\sqrt{y}}^{1-y} f(x, y) \, dx \, dy
\]

Lecture Problems

2. Find the volume of the tetrahedron bounded by the coordinate planes and \( 8x + 2y + 3z = 24 \).

Solution: 48

3. Find the volume of the tetrahedron bounded by the coordinate planes and \( -x + 2y - 3z = 24 \).

Solution: Remember to graph the tetrahedron! 384

4. Find the volume of the solid in the first octant bounded by the coordinate planes and the planes \( 2x + y - 4 = 0 \) and \( 8x + y - 4z = 0 \).

Solution:

\[
\int_{0}^{3} \int_{0}^{-2x+4} \frac{1}{4} (8x + y) \, dy \, dx = \frac{20}{3}
\]

5. Find the volume of the solid in the first octant bounded by the cylinder \( y = x^2 \) and the planes \( x = 0 \), \( z = 0 \) and \( y + z = 1 \).

Solution:

\[
\int_{0}^{1} \int_{-y}^{y} 1 - y \, dy \, dx = \frac{4}{15}
\]
6. Find the volume of the solid bounded by \( z = e^{x-y} \), the plane \( x + y = 1 \) and the coordinate planes.

**Solution:**

\[
\int_0^1 \int_0^{1-x} e^{x-y} \, dy \, dx = \frac{e}{2} + \frac{1}{2e} - 1
\]

7. Find the volume of the solid bounded by the cylinders \( x^2 + z^2 = 16 \), \( y^2 + z^2 = 16 \) and the coordinate planes.