Name:
ID:
Section:

This exam has 20 multiple choice questions worth 5 points each.
Important:

- No graphing calculators! Any non-graphing scientific calculator is fine.
- For the multiple choice questions, mark your answer on the answer card.
- Show all your work for the written problems. You will be graded on the ease of reading your solution as well as for your work.
- You are allowed both sides of 3×5 note "cheat card" for the exam.

1. Find the equation of the plane through the point $(1, 2, 4)$ and parallel to the plane $2x+y-3z = 18$.

   (a) $2x + y - 3z = -18$
   (b) $2x + y - 3z = -8$
   (c) $2x + y - 3z = 8$
   (d) $2x + y - 3z = 18$
   (e) $x + 2y + 4z = -21$
   (f) $x + 2y + 4z = -18$
   (g) $x + 2y + 4z = 18$
   (h) $x + 2y + 4z = 21$

2. Find the vector projection of the vector $\mathbf{a} = (3, 0, 3)$ onto the vector $\mathbf{b} = (1, -1, 1)$.

   (a) $\left(\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}\right)$
   (b) $(1, -1, 1)$
   (c) $(2, -2, 2)$
   (d) $(1, 0, 1)$
   (e) $(3, 0, 3)$
   (f) $(6, 0, 6)$
   (g) $(4, -1, 4)$

3. Find the minimal distance from the origin to the plane $x + 5y - z = 6$.

   (a) 0
   (b) $\frac{2}{\sqrt{5}}$
   (c) $\frac{2}{5}$
   (d) $\frac{2}{\sqrt{3}}$
   (e) $\frac{2}{3}$
   (f) $\frac{2}{\sqrt{2}}$
   (g) 1

4. Which of the following sets in $\mathbb{R}$ are closed?

    I. $[0, 3]$
    II. $\mathbb{R}$
    III. $[-1, 1)$
    IV. $\mathbb{Q}$, the set of all rational numbers (fractions)

    (a) I only
    (b) II only
    (c) III only
    (d) IV only
    (e) I and II only
    (f) I and III only
    (g) I and IV only
    (h) II and III only
    (i) III and IV only
    (j) Exactly three are closed
    (k) Some other combination

5. Which best describes the spherical equation $\rho = 8\cos\theta\sin\phi$?

    (a) A sphere centered at the origin
    (b) A sphere, not centered at the origin
    (c) A cone
    (d) A plane
    (e) A paraboloid
    (f) A saddle
    (g) None of the above

6. Find where the tangent plane to the surface $x^2 - y^3 + z^4 = 4$ at the point $(2, 1, 1)$ hits the y-axis.

   (a) -3
   (b) -2
   (c) -1
   (d) 0
   (e) 1
   (f) 2
   (g) 3
   (h) 4
   (i) 5
   (j) 6

7. Let $f(x, y) = x^3y$. Use a linearization at $(1, 2)$ to approximate $f(1.2, 1.3)$.

   (a) 2.0
   (b) 2.1
   (c) 2.2
   (d) 2.3
   (e) 2.4
   (f) 2.5
   (g) 2.6
   (h) 2.7
   (i) 2.8
   (j) 2.9

8. Find the directional derivative of $f(x, y) = x^3y - xy^2$ at the point $(1, 3)$ in the direction of the vector $\langle -3, 4 \rangle$.

   (a) -8
   (b) -7
   (c) -6
   (d) -5
   (e) -4
   (f) -3
   (g) -2
   (h) -1
   (i) 0
   (j) 1

9. Find the maximum of $f(x, y) = x^2 + x + y^2$ on the disk $x^2 + y^2 \leq 1$.

   (a) 3
   (b) -2
   (c) -1
   (d) $-\frac{1}{2}$
   (e) $-\frac{1}{4}$
   (f) 0
   (g) $\frac{1}{4}$
   (h) $\frac{1}{2}$
   (i) 1
   (j) 2
   (k) 3

10. The function $f(x, y) = x^4 + y^4 - 16xy - 2$ has 3 critical points: $(-2, -2)$, $(0, 0)$, and $(2, 2)$. These critical points are

 (a) 3 local maximums
 (b) 2 local maximums, 1 local minimum
 (c) 2 local maximums, 1 saddle
 (d) 1 local maximum, 2 local minimums
 (e) 1 local maximum, 1 local minimum, 1 saddle
 (f) 1 local maximum, 2 saddles
 (g) 3 local minimums
 (h) 2 local minimums, 1 saddle
 (i) 1 local minimum, 2 saddles
 (j) 3 saddles

11. Let C be the boundary of the triangle with vertices $(1, 0)$, $(0, 1)$ and $(-1, -1)$ oriented counter-clockwise. Let

$$G = \left(\frac{-y}{x^2 + y^2}, \frac{x}{x^2 + y^2}\right)$$

Compute $\displaystyle\int_C G \cdot dr$

 (a) -2π
 (b) $-\pi$
 (c) 0
 (d) $\dfrac{\pi}{2}$
 (e) π
 (f) $\dfrac{3\pi}{2}$
 (g) 2π
 (h) 3π

12. Compute

$$\int_C (2x + 3y)\, dx + (3x - 8y)\, dy$$

where C is the union of the line segments from $(2, 3)$ to $(-2, 4)$ and then to $(1, 1)$.

(a) 6
(b) 7
(c) 8
(d) 9
(e) 10
(f) 11
(g) 12
(h) 13
(i) 14
(j) 15

13. Let C be the curve with parametrization $r(t) = (t, t^2)$ for $0 \le t \le 2$. Compute $\int_C 3xy\, dx - 2x^2\, dy$

(a) -9
(b) -8
(c) -7
(d) -6
(e) -5
(f) -4
(g) -3
(h) -2
(i) -1
(j) 0

14. Let C be the boundary of the square with vertices $(0,0)$, $(2,0)$, $(2,2)$, $(0,2)$ oriented counter-clockwise. Find $\int_C x^2\,dx + xy\,dy$.

    (a) 1
    (b) 2
    (c) 3
    (d) 4
    (e) 5
    (f) 6
    (g) 7
    (h) 8
    (i) 9
    (j) 10

15. Let $f(x,y,z)$ be a smooth function. Let $C(t)$ be a smooth curve in $\mathbb{R}^3$, $0 \le t \le 1$, be completely contained in the level surface $f(x,y,z) = 5$. What is

$$\int_C \nabla f \cdot dr$$

    (a) -2π
    (b) -5
    (c) 0
    (d) 5
    (e) 2π
    (f) Can not determine with the given information

16. Compute the average value of $f(x, y) = x + 2y$ over the box $[0, 2] \times [0, 2]$

   (a) 0
   (b) 1
   (c) 2
   (d) 3
   (e) 4
   (f) 6
   (g) 8
   (h) 10
   (i) 12

17. A solid portion of the spherical shell lies in the first octant between $x^2 + y^2 + z^2 = 4$ and $x^2 + y^2 + z^2 = 9$. The density of the solid has density function $\delta(x, y, z) = \dfrac{1}{x^2 + y^2 + z^2}$.

   Find the mass of the solid.

   (a) 0
   (b) $\frac{\pi}{4}$
   (c) $\frac{\pi}{2}$
   (d) π
   (e) 2π
   (f) 3π

18. A tetrahedron with vertices $(0,0,0)$, $(1,0,0)$, $(1,1,0)$, and $(0,1,1)$ has density $\delta(x,y,z)=1$. Compute the mass of the tetrahedron.

(a) $\frac{1}{12}$

(b) $\frac{1}{10}$

(c) $\frac{1}{9}$

(d) $\frac{1}{8}$

(e) $\frac{1}{6}$

(f) $\frac{1}{5}$

(g) $\frac{1}{4}$

(h) $\frac{1}{3}$

(i) $\frac{1}{2}$

(j) 1

19. Let R be the region in $\mathbb{R}^3$ bounded by the cylinder $x^2+y^2=9$, the planes $z=-1$ and $z=x+3$. Find $\displaystyle\iiint_R \sqrt{x^2+y^2}\,dV$.

   (a) 0
   (b) 9π
   (c) 18π
   (d) 27π
   (e) 36π
   (f) 45π
   (g) 54π
   (h) 63π
   (i) 72π
   (j) 90π

20. Which of the following integrals represent the volume of the portion of the unit sphere centered at the origin that lies in the first octant?

   I. $\displaystyle\int_0^1\int_0^{\sqrt{1-y^2}} \sqrt{1-x^2-y^2}\,dx\,dy$

   II. $\displaystyle\int_0^{\frac{\pi}{2}}\int_0^1 r\sqrt{1-r^2}\,dr\,d\theta$

   III. $\displaystyle\int_0^{\frac{\pi}{2}}\int_0^1\int_0^{\sqrt{1-r^2}} r\,dz\,dr\,d\theta$

   IV. $\displaystyle\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}}\int_0^1 \rho^2\sin\phi\,d\rho\,d\phi\,d\theta$

   (a) Exactly one of these
   (b) I and II only
   (c) I and III only
   (d) I and IV only
   (e) I, II and III only
   (f) I, II and IV only
   (g) I, III and IV only
   (h) II, III and IV only
   (i) All
   (j) None