Name:  
ID:  
Section:  
This exam has 17 questions:

- 15 multiple choice questions worth 5 points each.
- 2 hand graded questions worth 25 points total.

Important:

- No graphing calculators! Any non-graphing scientific calculator is fine.
- For the multiple choice questions, mark your answer on the answer card.
- Show all your work for the written problems. You will be graded on the ease of reading your solution as well as for your work.
- You are allowed both sides of 3 × 5 note “cheat card” for the exam.

1. Let \( r(t) \) be the line from \((1, 2)\) to \((4, -2)\). Compute

\[
\int_{r} y^2 \, dx - x \, dy
\]

(a) Something less than 8  
(b) 8  
(c) 9  
(d) 10  
(e) 11  
(f) 12  
(g) 13  
(h) 14  
(i) 15  
(j) Something greater than 15  
(k) Unable to determine with the given information
2. Compute the work done by the vector field \( F = (y^2, x^2, 2z) \) along a curve that travels from the origin to \((1, 2, 5)\).

(a) Something less than 25
(b) 25
(c) 26
(d) 27
(e) 28
(f) 29
(g) 30
(h) 31
(i) 32
(j) Something greater than 32
(k) Unable to determine with the given information

3. Compute the work done by the vector field \( F = (y, x, 2) \) along a curve that travels from the origin to \((1, 2, 5)\).

(a) Something less than 5
(b) 5
(c) 6
(d) 7
(e) 8
(f) 9
(g) 10
(h) 11
(i) 12
(j) Something greater than 12
(k) Unable to determine with the given information
4. Let $C$ be the boundary of the rectangle $R = [1, 2] \times [0, 3]$ where $C$ is parametrized counterclockwise. Compute

$$
\int_C (e^{x^2} - x^2 y) \, dx + (x + \sin y^3) \, dy
$$

(a) Something less than 10  
(b) 10  
(c) 11  
(d) 12  
(e) 13  
(f) 14  
(g) 15  
(h) 16  
(i) 17  
(j) 18  
(k) Something greater than 18

5. Let

$$
F = \left( \frac{x - y}{x^2 + y^2}, \frac{x + y}{x^2 + y^2} \right)
$$

Let $r(t)$ be the positively oriented (counter-clockwise) unit circle. Compute

$$
\frac{1}{2\pi} \int_r F \cdot dr
$$

(a) $-3$  
(b) $-2$  
(c) $-1$  
(d) 0  
(e) 1  
(f) 2  
(g) 3  
(h) Integral is undefined
6. Let $C(t)$ be the curve traced out, in the counter-clockwise direction, along the triangle with vertices $(-4, -2)$, $(2, -2)$ and $(1, 2)$. Compute

$$\int_C 3y \, dx + 4x \, dy$$

(a) 7
(b) 8
(c) 9
(d) 10
(e) 11
(f) 12
(g) 13
(h) 14
(i) 15
(j) 16
(k) 17

7. Suppose $F = (f, g)$ is a differentiable vector field defined on an open subset $U \subset \mathbb{R}^2$ with

$$\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y} = 0$$

Which of the following are true?

I. Let $C(t)$ be a closed path (a loop) in $U$. Then $\int_C F \cdot dr = 0$.

II. Let $C(t)$ be a path in $U$ with $C^{-}$ the reverse path of $C$. Then $\int_C F \cdot dr + \int_{C^{-}} F \cdot dr = 0$

III. Let $C(t)$ be a closed path (a loop) in $U$ and let $R$ be some rectangle completely contained in $U$. If $C$ is completely contained in $R$ then $\int_C F \cdot dr = 0$.

(a) I only
(b) II only
(c) III only
(d) I and II only
(e) I and III only
(f) II and III only
(g) All are true
(h) None are true
8. The region, $R$, for the integral below is an isosceles triangle.

$$\int\int_R f(x, y) \, dA = \int_{-3}^{-1} \int_0^{y+3} f(x, y) \, dx \, dy + \int_{-1}^{1} \int_0^{1-y} f(x, y) \, dx \, dy$$

Choose the best triangle below representing the region of integration.
9. Which of the following integrals represents the volume of the solid lying between \( x = 1 \) and \( x = 5 - y^2 \), above \( z = 0 \), and below \( z = x^2 + y \).

(a) \( \int_0^2 \int_1^{5-x^2} x^2 + y \ dy \ dx \)
(b) \( \int_0^2 \int_1^{5-y^2} x^2 + y \ dx \ dy \)
(c) \( \int_{-2}^2 \int_1^{5-x^2} x^2 + y \ dy \ dx \)
(d) \( \int_{-2}^2 \int_1^{5-y^2} x^2 + y \ dx \ dy \)
(e) \( \int_0^2 \int_1^{x^2+y} 5 - y^2 \ dy \ dx \)
(f) \( \int_0^2 \int_1^{x^2+y} 5 - y^2 \ dx \ dy \)
(g) None of the above

10. The following

\[
\int_{-5}^{-5} \int_{\sqrt{4-y}}^{\sqrt{4-y}/2} f(x, y) \ dx \ dy + \int_{3}^{4} \int_{\sqrt{4-y}}^{-\sqrt{4-y}} f(x, y) \ dx \ dy
\]

is equivalent to an integral of the form

\[
\int_{a}^{b} \int_{A(x)}^{B(x)} f(x, y) \ dy \ dx
\]

What is

\( b - a + B(1) - A(1) \)

(a) 5
(b) 6
(c) 7
(d) 8
(e) 9
(f) 10
(g) 11
(h) 12
(i) 13
(j) 14
(k) 15
11. Compute
\[ \int_0^{\sqrt{\pi}} \int_{y/2}^{\sqrt{\pi}/2} \sin(x^2) \, dx \, dy \]
(a) 1
(b) \(-1\)
(c) \(\frac{3}{2}\)
(d) \(\frac{1}{2}\)
(e) \(1 + \frac{1}{\sqrt{2}}\)
(f) \(1 - \frac{1}{\sqrt{2}}\)
(g) \(1 + \frac{\sqrt{3}}{2}\)
(h) \(1 - \frac{\sqrt{3}}{2}\)

12. Which of the following integrals is positive (strictly greater than 0)?

I. \[ \int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} x \, dy \, dx \]

II. \[ \int_{-3}^{0} \int_{-1}^{7} x \, dx \, dy \]

III. \[ \int_{-3}^{0} \int_{-1}^{7} x \, dy \, dx \]

IV. \[ \int_{0}^{2} \int_{-\sqrt{1-(x-1)^2}}^{\sqrt{1-(x-1)^2}} x \, dy \, dx \]

(a) I only
(b) II only
(c) III only
(d) IV only
(e) I and II only
(f) I and III only
(g) I and IV only
(h) II and III only
(i) II and IV only
(j) III and IV only
(k) Some other combination or none are positive
13. Select the polar form of the equation $x^2 = y^3$.

(a) $r = \tan \theta$
(b) $r = (\tan \theta)^{3/2}$
(c) $r = (\tan \theta)^{2/3}$
(d) $r = \cos \theta$
(e) $r = (\cos \theta)^{3/2}$
(f) $r = (\cos \theta)^{2/3}$
(g) $r = \sin \theta$
(h) $r = (\sin \theta)^{3/2}$
(i) $r = (\sin \theta)^{2/3}$
(j) $r = \tan^2 \theta \sec \theta$
(k) $r = \cot^2 \theta \csc \theta$

14. Compute $\int_\mathbb{R} \frac{1}{(x^2 + y^2)^{3/2}} \ dA$ where $R$ is the annulus $R = \{(x, y) | 1 \leq x^2 + y^2 \leq 4\}$

(a) 0
(b) $\frac{\pi}{4}$
(c) $\frac{\pi}{2}$
(d) $\frac{3\pi}{4}$
(e) $\pi$
(f) $\frac{5\pi}{4}$
(g) $\frac{3\pi}{2}$
(h) $\frac{7\pi}{4}$
(i) $2\pi$
15. Compute

\[ \int_0^{\pi/4} \int_0^{\tan \theta \sec \theta} 5r^2 \sin \theta \, dr \, d\theta + \int_0^{\pi/4} \int_0^{\csc \theta} 5r^2 \sin \theta \, dr \, d\theta \]

(a) Something less than 0
(b) 1
(c) 2
(d) 3
(e) 4
(f) 5
(g) 6
(h) 7
(i) 8
(j) 9
(k) Something greater than 9
16. Compute the volume of the ice cream cone shaped solid bounded above by the hemisphere: \( z = 1 + \sqrt{1 - x^2 - y^2} \) and bounded below by the cone: \( z = \sqrt{x^2 + y^2} \).
17. Let $F$ be the vector field in the plane given by $F(x,y) = (x - y^3, x^3 - y)$. Let $R$ be the half disc defined by $x \geq 0$ and $x^2 + y^2 \leq 1$. Let $C$ be the boundary of $R$, parametrized positively (counter-clockwise).

(a) Draw the region $R$.
(b) Write down a parametrization of $C$.
(c) Write down, using Green’s Theorem, a double integral that is equal to

$$\int_C F \cdot dr$$

(d) Compute, by any means of your choice

$$\int_C F \cdot dr$$