Math 233 - December 4, 2009

• Center of Mass and Moments

\[ M = \iint_R \delta(x, y) \, dA \]
\[ M_y = \iint_R x \delta(x, y) \, dA \]
\[ M_x = \iint_R y \delta(x, y) \, dA \]
\[ \bar{x} = \frac{M_y}{M} \]
\[ \bar{y} = \frac{M_x}{M} \]
\[ I_z = \iint_R r^2 \delta(x, y) \, dA \]

1. Let \( R \) be the “ice cream cone” inside the sphere \( \rho = 1 \) and inside the cone \( \phi = \pi/6 \) (with \( z \geq 0 \)). Suppose that \( \delta(x, y, z) = z \). Compute the integrals:

\[ M = \iiint_R \delta(x, y, z) \, dV = \]
\[ M_{yz} = \iiint_R x \delta(x, y, z) \, dV = \]
\[ M_{xz} = \iiint_R y \delta(x, y, z) \, dV = \]
\[ M_{yz} = \iiint_R z \delta(x, y, z) \, dV = \]

Lecture Problems

2. Find the center of mass of the tetrahedron with vertices \((0, 0, 0), (3, 0, 0), (0, 3, 0) \) and \((0, 0, 3) \) and with density \( \delta(x, y, z) = x + y + z + 1 \).

3. Let \( R \) be the region bounded by \( x^2 + y^2 = z^2 \) and \( x^2 + y^2 = 4y \). Set up the integral using spherical coordinates.

\[ \iiint_R \frac{1}{x^2 + y^2 + z^2} \, dV = \]