Math 233 - December 4, 2009

- Center of Mass and Moments

$$
\begin{aligned}
M &= \iint_R \delta(x,y)\, dA \\
M_y &= \iint_R x\, \delta(x,y)\, dA \\
M_x &= \iint_R y\, \delta(x,y)\, dA \\
\overline{x} &= \frac{M_y}{M} \\
\overline{y} &= \frac{M_x}{M} \\
I_z &= \iint_R r^2 \delta(x,y)\, dA
\end{aligned}
$$

1. Let R be the "ice cream cone" inside the sphere $\rho = 1$ and inside the cone $\phi = \pi/6$ (with $z \geq 0$). Suppose that $\delta(x,y,z) = z$. Compute the integrals:

$$
\begin{aligned}
M &= \iiint_R \delta(x,y,z)\, dV = \\
M_{yz} &= \iiint_R x\delta(x,y,z)\, dV = \\
M_{xz} &= \iiint_R y\delta(x,y,z)\, dV = \\
M_{yz} &= \iiint_R z\delta(x,y,z)\, dV =
\end{aligned}
$$

Lecture Problems

2. Find the center of mass of the tetrahedron with vertices $(0,0,0)$, $(3,0,0)$, $(0,3,0)$ and $(0,0,3)$ and with density $\delta(x,y,z) = x + y + z + 1$.

3. Let R be the region bounded by $x^2+y^2 = z^2$ and $x^2+y^2 = 4y$. Set up the integral using spherical coordinates.

$$
\iiint_R \frac{1}{x^2+y^2+z^2}\, dV =
$$