Math 233 - December 3, 2009

- Spherical transformation of integrals

1. Compute

$$\int_a^\rho \pi(\rho^2 - z^2)\,dz =$$

2. Let $f(t)$ be a function on the inteval $[a, b]$. What does the Mean Value Theorem say about f?
3. Let $f(\rho) = \frac{1}{3}\rho^3$ on the interval $[\rho_1, \rho_2]$. What does the mean value theorem say about this f?
4. Let $f(\phi) = \cos\phi$ on the interval $[\phi_1, \phi_2]$. What does the mean value theorem say about this f?

1. Compute
$$\int_a^\rho \pi(\rho^2 - z^2)\, dz = \pi\left(\frac{2}{3}\rho^3 - a\rho^2 + \frac{1}{3}a^3\right)$$
2. Let $f(t)$ be a function on the inteval $[a, b]$. What does the Mean Value Theorem say about f?
   Solution: There is some $c \in [a, b]$ such that $f(b) - f(a) = f'(c)(b - a)$.
3. Let $f(\rho) = \frac{1}{3}\rho^3$ on the interval $[\rho_1, \rho_2]$. What does the mean value theorem say about this f?
   Solution: There is some $\overline{\rho} \in [\rho_1, \rho_2]$ such that $f(\rho_2) - f(\rho_1) = f'(\overline{\rho})(\rho_2 - \rho_1)$ or
$$\frac{\rho_2^3}{3} - \frac{\rho_1^3}{3} = \overline{\rho}^2(\rho_2 - \rho_1)$$
4. Let $f(\phi) = \cos\phi$ on the interval $[\phi_1, \phi_2]$. What does the mean value theorem say about this f?
   Solution: There is some $\overline{\phi} \in [\phi_1, \phi_2]$ such that $f(\phi_2) - f(\phi_1) = f'(\overline{\phi})(\phi_2 - \phi_1)$ or
$$\cos\phi_1 - \cos\phi_2 = \sin\overline{\phi}(\phi_2 - \phi_1)$$

Lecture Problems

5.

$$\int_0^3 \int_0^{\sqrt{9-y^2}} \int_{x^2+y^2}^{\sqrt{18-x^2-y^2}} (x^2+y^2+z^2)\, dz\, dx\, dy$$

Lecture Problems

5.

$$\int_0^3 \int_0^{\sqrt{9-y^2}} \int_{x^2+y^2}^{\sqrt{18-x^2-y^2}} (x^2+y^2+z^2)\, dz\, dx\, dy$$

$$= \int_0^{\pi/4} \int_0^{\pi/2} \int_0^{3\sqrt{2}} \rho^4 \sin\phi\, d\rho\, d\theta\, d\phi = \frac{486\pi}{5}(\sqrt{2}-1)$$

6. Let R be the region bounded by $x^2 + y^2 = z^2$ and $x^2 + y^2 = 4y$. Set up the integral using spherical coordinates.

$$\iiint_R \frac{1}{x^2 + y^2 + z^2} \, dV =$$

6. Let R be the region bounded by $x^2 + y^2 = z^2$ and $x^2 + y^2 = 4y$. Set up the integral using spherical coordinates.

$$\iiint_R \frac{1}{x^2 + y^2 + z^2}\, dV =$$

$$\int_{\pi/4}^{3\pi/4} \int_0^{\pi} \int_0^{4 \sin\theta / \sin\phi} \sin\phi \; d\rho \; d\theta \; d\phi = 4\pi$$