Math 233 - December 3, 2009

• Spherical transformation of integrals

1. Compute
\[ \int_0^\rho \pi (\rho^2 - z^2) \, dz = \]

2. Let \( f(t) \) be a function on the interval \([a, b] \). What does the Mean Value Theorem say about \( f \)?

3. Let \( f(\rho) = \frac{1}{3} \rho^3 \) on the interval \([\rho_1, \rho_2] \). What does the mean value theorem say about this \( f \)?

4. Let \( f(\phi) = \cos \phi \) on the interval \([\phi_1, \phi_2] \). What does the mean value theorem say about this \( f \)?

Lecture Problems

5.
\[ \int_0^3 \int_0^{\sqrt{9-y^2}} \int_{x^2+y^2}^{\sqrt{18-x^2-y^2}} (x^2 + y^2 + z^2) \, dz \, dx \, dy = \]

6. Let \( R \) be the region bounded by \( x^2 + y^2 = z^2 \) and \( x^2 + y^2 = 4y \). Set up the integral using spherical coordinates.
\[ \int \int \int_{R} \frac{1}{x^2 + y^2 + z^2} \, dV = \]