Math 233 - December 1, 2009
Solutions

- Spherical coordinates

1. Find the determinant:

\[
\begin{vmatrix}
\sin \phi \cos \theta & \rho \cos \phi \cos \theta & -\rho \sin \phi \sin \theta \\
\sin \phi \sin \theta & \rho \cos \phi \sin \theta & \rho \sin \phi \cos \theta \\
\cos \phi & -\rho \sin \phi & 0
\end{vmatrix} = \rho^2 \sin \phi
\]

Lecture Problems

2. Convert the spherical equation to a Cartesian equation: \( \rho = \sec \phi \)
   
   Solution: \( z = 1 \)

3. Convert the Cartesian equation to a spherical equation:
   \( x^2 + y^2 + 4z^2 = 10 \)
   
   Solution: \( \rho = (3/2) \sec \phi \)

4. Convert the Cartesian equation to a spherical equation:
   \( x^2 + y^2 - 2z^2 = 0 \)
   
   Solution: \( \tan \phi = 1/\sqrt{2} \)

5. Convert the Cartesian equation to a spherical equation:
   \( x + y + z = 1 \)
   
   Solution: \( \rho \sin \phi \cos \theta + \rho \sin \phi \sin \theta + \rho \cos \phi = 1 \)

6. Convert the spherical equation to a Cartesian equation: \( \rho \sin \phi = 1 \)
   
   Solution: \( x^2 + y^2 = 1 \)

7. Find the volume of the ice cream cone—the solid below the sphere \( x^2 + y^2 + z^2 = 1 \) and above the cone \( z^2 = x^2 + y^2 \).
   
   \[
   V = \int_0^{2\pi} \int_0^{\pi/4} \int_0^1 \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta = \frac{\pi(2 - \sqrt{2})}{3}
   \]

8. Find the mass of the solid inside the sphere \( \rho = 3 \) and outside the sphere \( \rho = 2 \) with density equal to the distance from the origin.
   
   \[
   m = \int_0^{2\pi} \int_0^\pi \int_2^3 \rho \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta = 65\pi
   \]

9.

\[
\int_{-3}^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_{-\sqrt{9-x^2-z^2}}^{\sqrt{9-x^2-z^2}} (x^2 + y^2 + z^2)^{3/2} \, dy \, dz \, dx
\]

\[
= \int_2^{2\pi} \int_0^\pi \int_0^3 \rho^3 \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta = 486\pi
\]