Math 233 - December 1, 2009

- Spherical coordinates

1. Find the determinant:

$$\begin{vmatrix} \sin\phi\,\cos\theta & \rho\cos\phi\,\cos\theta & -\rho\sin\phi\,\sin\theta \\ \sin\phi\,\sin\theta & \rho\cos\phi\,\sin\theta & \rho\sin\phi\,\cos\theta \\ \cos\phi & -\rho\sin\phi & 0 \end{vmatrix} =$$

1. Find the determinant:

$$\begin{vmatrix} \sin\phi\,\cos\theta & \rho\cos\phi\,\cos\theta & -\rho\sin\phi\,\sin\theta \\ \sin\phi\,\sin\theta & \rho\cos\phi\,\sin\theta & \rho\sin\phi\,\cos\theta \\ \cos\phi & -\rho\sin\phi & 0 \end{vmatrix} = \rho^2\sin\phi$$

Lecture Problems

2. Convert the spherical equation to a Cartesian equation: $\rho = \sec\phi$
3. Convert the Cartesian equation to a spherical equation: $x^2 + y^2 + 4z^2 = 10$
4. Convert the Cartesian equation to a spherical equation: $x^2 + y^2 - 2z^2 = 0$
5. Convert the Cartesian equation to a spherical equation: $x + y + z = 1$
6. Convert the spherical equation to a Cartesian equation: $\rho \sin\phi = 1$

Lecture Problems

2. Convert the spherical equation to a Cartesian equation: $\rho = \sec\phi$
   Solution: $z = 1$
3. Convert the Cartesian equation to a spherical equation:
   $x^2 + y^2 + 4z^2 = 10$
   Solution: $\rho = (3/2)\sec\phi$
4. Convert the Cartesian equation to a spherical equation:
   $x^2 + y^2 - 2z^2 = 0$
   Solution: $\tan\phi = 1/\sqrt{2}$
5. Convert the Cartesian equation to a spherical equation:
   $x + y + z = 1$
   Solution: $\rho\sin\phi\cos\theta + \rho\sin\phi\sin\theta + \rho\cos\phi = 1$
6. Convert the spherical equation to a Cartesian equation: $\rho\sin\phi = 1$
   Solution: $x^2 + y^2 = 1$

7. Find the volume of the ice cream cone–the solid below the sphere $x^2 + y^2 + z^2 = 1$ and above the cone $z^2 = x^2 + y^2$.

$$V =$$

8. Find the mass of the solid inside the sphere $\rho = 3$ and outside the sphere $\rho = 2$ with density equal to the distance from the origin.

$$m =$$

9.

$$\int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_{-\sqrt{9-x^2-z^2}}^{\sqrt{9-x^2-z^2}} (x^2 + y^2 + z^2)^{3/2}\, dy\, dz\, dx$$

7. Find the volume of the ice cream cone–the solid below the sphere $x^2 + y^2 + z^2 = 1$ and above the cone $z^2 = x^2 + y^2$.

$$V = \int_0^{2\pi} \int_0^{\pi/4} \int_0^1 \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta = \frac{\pi(2-\sqrt{2})}{3}$$

8. Find the mass of the solid inside the sphere $\rho = 3$ and outside the sphere $\rho = 2$ with density equal to the distance from the origin.

$$m = \int_0^{2\pi} \int_0^{\pi} \int_2^3 \rho\rho^2 \sin\phi \, d\rho \, d\phi \, d\theta = 65\pi$$

9.

$$\int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_{-\sqrt{9-x^2-z^2}}^{\sqrt{9-x^2-z^2}} (x^2+y^2+z^2)^{3/2} \, dy \, dz \, dx$$
$$= \int_2^{2\pi} \int_0^{\pi} \int_0^3 \rho^{3/2}\rho^2 \sin\phi \, d\rho \, d\phi \, d\theta = 486\pi$$