Math 233 Warm Up Problems

August 28, 2009

1. Find the angle (in radians) between the two vectors in $\mathbb{R}^5$:

$$v_1 = (-1, 1, -2, 3), \qquad v_2 = (3, 2, -1, 2)$$

2. Find the angles of the triangle with vertices

$$p_1 = (1, 2, 3), \quad p_2 = (2, 1, 3), \quad p_3 = (3, 1, 2)$$

1. Find the angle (in radians) between the two vectors in $\mathbb{R}^5$:

$$v_1 = (-1, 1, -2, 3), \qquad v_2 = (3, 2, -1, 2)$$

Solution:

$$\cos\theta = \frac{v_1 \cdot v_2}{\|v_1\|\|v_2\|} = \frac{7}{3\sqrt{30}} \approx 0.42601$$

$$\theta = \cos^{-1}\left(\frac{7}{3\sqrt{30}}\right) \approx 1.13072$$

2. Find the angles of the triangle with vertices

$$p_1 = (1, 2, 3), \quad p_2 = (2, 1, 3), \quad p_3 = (3, 1, 2)$$

Solution: The angle at vertex $(1, 2, 3)$ is determined by the vectors $v_{12} = p_2 - p_1 = (1, -1, 0)$ and $v_{13} = p_3 - p_1 = (2, -1, -1)$. Computing $\cos\theta$:

$$\cos\theta = \frac{v_1 \cdot v_2}{\|v_1\|\|v_2\|} = \frac{3}{2\sqrt{3}} \approx 0.8660254$$

$$\theta = \cos^{-1}\left(\frac{3}{2\sqrt{3}}\right) \approx 0.5235988$$

Lecture Problems

3. Find a parametric equation of the line through the points

$$(1, 2, -8), (9, 7, 1)$$

4. Consider the parametric line

$$X(t) = (4 + 2t, 5 - 7t, -2 - t)$$

   (a) Find two points on this line.
   (b) Find the direction vector of this line.

Lecture Problems

3. Find a parametric equation of the line through the points

$$(1, 2, -8), (9, 7, 1)$$

Solution: There are many solutions, but here are what are probably the most common answers.

$$X(t) = (1, 2, 8) + t(8, 5, 9)$$
$$X(t) = (9, 7, 1) + t(8, 5, 9)$$

4. Consider the parametric line

$$X(t) = (4 + 2t, 5 - 7t, -2 - t)$$

   (a) Find two points on this line.
   Solution: There are lots of correct answers here but here are two I choose:

$$(4, 5, -2), (6, -2, -3)$$

   (b) Find the direction vector of this line.
   Solution: $v = (2, -7, -1)$

5. Find the distance between the point $P = (4, 2, -1)$ and the line $X(t) = (1, 2, -1) + t(2, -1, 3)$.

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Solution: You really need to draw the picture to understand this. We need to find a vector, orthogonal to $v = (2, -1, 3)$, going through the line and the point P. We do this using vector projections.
Let $Q = (1, 2, -1)$, notice that Q is on the line. Let $u = \overrightarrow{QP} = P - Q$. If we project u to the vector v we get

$$\mathrm{Pr}_v u = \left(\frac{6}{7}, -\frac{3}{7}, \frac{9}{7}\right)$$

The vector we really want is

$$N = u - \mathrm{Pr}_v u = \left(\frac{15}{7}, \frac{3}{7}, -\frac{9}{7}\right)$$

and the distance is

$$\|N\| = \frac{\sqrt{45}}{7}$$

You can also do this using 1 variable calculus, but this is simpler.