Math 233 Warm Up Problems

August 28, 2009

1. Find the equation of the sphere in $\mathbb{R}^3$ with radius π and center at $(\sqrt{2}, -7, \sqrt{\pi})$.
2. Find the unit vector in the direction of the given vector

$$V = (4, -2, 7) \qquad U =$$

1. Find the equation of the sphere in $\mathbb{R}^3$ with radius π and center at $(\sqrt{2}, -7, \sqrt{\pi})$.
   Solution:

$$(x - \sqrt{2})^2 + (y + 7)^2 + (z - \sqrt{\pi})^2 = \pi^2$$

2. Find the unit vector in the direction of the given vector

$$V = (4, -2, 7) \qquad U = \left(\frac{4}{\sqrt{69}}, -\frac{2}{\sqrt{69}}, \frac{7}{\sqrt{69}} \right)$$

Lecture Problems

3. Let $A = (12, -1, 5)$ and $B = (-1, 2, 1)$.
   (a) Find the component of A along B.
   (b) Find the vector projection of A along B.
   (c) Find the scalar projection of A along B.
   (d) Find the orthogonal projection of A along B.

Lecture Problems

3. Let $A = (12, -1, 5)$ and $B = (-1, 2, 1)$.

   (a) Find the component of A along B.

$$c = \frac{A \cdot B}{B \cdot B} = -\frac{3}{2}$$

   (b) Find the vector projection of A along B.

$$\mathrm{Pr}_B A = cB = \left(\frac{3}{2}, -3, -\frac{3}{2}\right)$$

   (c) Find the scalar projection of A along B.

$$\|\mathrm{Pr}_B A\| = \sqrt{\frac{27}{2}}$$

   (d) Find the orthogonal projection of A along B.

$$A - \mathrm{Pr}_B A = A - \frac{A \cdot B}{B \cdot B} B = \left(\frac{21}{2}, 2, \frac{13}{2}\right)$$

4. Let $A = (8, 1, -2)$ and $B = (1, 4, -3)$.
   (a) Find the component of A along B.
   (b) Find the vector projection of A along B.
   (c) Find the scalar projection of A along B.
   (d) Find the orthogonal projection of A along B.

4. Let $A = (8, 1, -2)$ and $B = (1, 4, -3)$.

   (a) Find the component of A along B.

$$c = \frac{A \cdot B}{B \cdot B} = \frac{9}{13}$$

   (b) Find the vector projection of A along B.

$$\mathrm{Pr}_B A = cB = \left(\frac{9}{13}, \frac{36}{13}, -\frac{27}{13}\right)$$

   (c) Find the scalar projection of A along B.

$$\|\mathrm{Pr}_B A\| = \frac{9\sqrt{2}}{\sqrt{13}}$$

   (d) Find the orthogonal projection of A along B.

$$A - \mathrm{Pr}_B A = A - \frac{A \cdot B}{B \cdot B} B = \left(\frac{95}{13}, -\frac{23}{13}, \frac{1}{13}\right)$$

5. Find the angle (in radians) between the two vectors in $\mathbb{R}^4$:

$$v_1 = (1, -2, -3, 1), \qquad v_2 = (4, 5, -1, 2)$$

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Solution:

$$\cos\theta = \frac{A \cdot B}{\|A\|\|B\|} = \frac{-1}{\sqrt{15}\sqrt{46}}$$

$$\theta = \cos^{-1}\left(-\frac{1}{\sqrt{690}}\right) \approx 1.608875$$