Math 233 Warm Up Problems
August 28, 2009
1. Find the equation of the sphere in $\mathbb{R}^3$ with radius $\pi$ and center at $(\sqrt{2}, -7, \sqrt{\pi})$.

2. Find the unit vector in the direction of the given vector

\[ V = (4, -2, 7) \quad U = \]
1. Find the equation of the sphere in \( \mathbb{R}^3 \) with radius \( \pi \) and center at \((\sqrt{2}, -7, \sqrt{\pi})\).

Solution:

\[
(x - \sqrt{2})^2 + (y + 7)^2 + (z - \sqrt{\pi})^2 = \pi^2
\]

2. Find the unit vector in the direction of the given vector

\[ V = (4, -2, 7) \quad U = \left( \frac{4}{\sqrt{69}}, -\frac{2}{\sqrt{69}}, \frac{7}{\sqrt{69}} \right) \]
Lecture Problems

3. Let $A = (12, -1, 5)$ and $B = (-1, 2, 1)$.

   (a) Find the component of $A$ along $B$.
   (b) Find the vector projection of $A$ along $B$.
   (c) Find the scalar projection of $A$ along $B$.
   (d) Find the orthogonal projection of $A$ along $B$. 
Lecture Problems

3. Let \( A = (12, -1, 5) \) and \( B = (-1, 2, 1) \).

(a) Find the component of \( A \) along \( B \).

\[
c = \frac{A \cdot B}{B \cdot B} = -\frac{3}{2}
\]

(b) Find the vector projection of \( A \) along \( B \).

\[
\text{Pr}_B A = cB = \left( \frac{3}{2}, -3, -\frac{3}{2} \right)
\]

(c) Find the scalar projection of \( A \) along \( B \).

\[
\|\text{Pr}_B A\| = \sqrt{\frac{27}{2}}
\]

(d) Find the orthogonal projection of \( A \) along \( B \).

\[
A - \text{Pr}_B A = A - \frac{A \cdot B}{B \cdot B} B = \left( \frac{21}{2}, 2, \frac{13}{2} \right)
\]
4. Let $A = (8, 1, -2)$ and $B = (1, 4, -3)$.
   (a) Find the component of $A$ along $B$.
   (b) Find the vector projection of $A$ along $B$.
   (c) Find the scalar projection of $A$ along $B$.
   (d) Find the orthogonal projection of $A$ along $B$. 
4. Let $A = (8, 1, -2)$ and $B = (1, 4, -3)$.

(a) Find the component of $A$ along $B$.

$$c = \frac{A \cdot B}{B \cdot B} = \frac{9}{13}$$

(b) Find the vector projection of $A$ along $B$.

$$\text{Pr}_B A = cB = \left( \frac{9}{13}, \frac{36}{13}, -\frac{27}{13} \right)$$

(c) Find the scalar projection of $A$ along $B$.

$$\|\text{Pr}_B A\| = \frac{9\sqrt{2}}{\sqrt{13}}$$

(d) Find the orthogonal projection of $A$ along $B$.

$$A - \text{Pr}_B A = A - \frac{A \cdot B}{B \cdot B} B = \left( \frac{95}{13}, -\frac{23}{13}, \frac{1}{13} \right)$$
5. Find the angle (in radians) between the two vectors in $\mathbb{R}^4$:

$v_1 = (1, -2, -3, 1), \quad v_2 = (4, 5, -1, 2)$
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$v_1 = (1, -2, -3, 1), \quad v_2 = (4, 5, -1, 2)$

Solution:

$$\cos \theta = \frac{A \cdot B}{\|A\|\|B\|} = \frac{-1}{\sqrt{15}\sqrt{46}}$$

$$\theta = \cos^{-1} \left(-\frac{1}{\sqrt{690}}\right) \approx 1.608875$$