

Math 217 - September 21, 2010

Solutions

- Review. Exam 1: 1.1-2.4

1. Determine all a and b for which the equation has a unique solution. $y' = y^{1/3}, y(a) = b$
Solution: Anything is fine as long as $b \neq 0$.
2. Determine all a and b for which the equation has a unique solution. $y' + ty = t + \frac{1}{t}, y(a) = b$
Solution: Anything is fine as long as $a \neq 0$.
3. Solve $(x^3 + y/x) dx + (y^2 + \ln x) dy = 0$. **Solution:** Equation is exact.
4. Determine stability for $x' = (x + 2)(x + 1)(x - 1)(x - 2)$. **Solution:** Stable points: $x = -2, x = 1$. Unstable: $x = -1, x = 2$.
5. Solve $t^2 y' = ty + y^2$ **Solution:** $y = t/(C - \ln |t|)$.
6. Solve $xy' - 5y = 3xy^{7/3}$. **Solution:** Bernoulli transform ($v = y^{-4/3}$) it into the equation $3v' + 20v/x = -12$, which is first order linear.
7. Torricelli's law is $A(y)dy/dt = -k\sqrt{y}$. At time $t = 0$, a plug at the bottom a cylindrical tank 16 ft high is removed. After one hour, the water in the tank is 9 ft deep. After how many hours will the tank be empty? **Solution:** 4
8. Solve $x^2 y' + xy = 1, y(1) = 1$. **Solution:** $y = (1/t)(\ln |t| + 1)$
9. Draw a slopefield for $y' = y^2 - x$
10. Solve $y' = 1 + x + y + xy$ **Solution:** Separable, $\ln |1 + y| = x + x^3/3 + C$
11. Solve $(x^2 + 1)y' + 3xy = 6x, y(0) = 2$. **Solution:** $y = 2$ is an equilibrium solution.
12. Solve $xy^2 y' = y^3 + x^4$ (hint: use $v = y/x$) **Solution:** $y = x\sqrt[3]{3x + C}$
13. A cascade of two tanks is set up such that tank 1, initially contains 200 gallons of corn syrup and tank 2 initially contains 100 gallons of water. Suppose that water flows into tank 1 at a rate of 1 gallon per minute, that the solution in tank 1 flows from tank 1 into tank 2 at a rate also of 1 gallon per minute, and that the solution in tank two also exits tank two at a rate of 1 gallon per minute. Assume that the solutions are completely mixed instantaneously.
 - (a) Find the amount $x(t)$ of corn syrup in tank 1 at time $t > 0$. **Solution:** $x(t) = 200e^{-t/200}$
 - (b) Find the amount $y(t)$ of corn syrup in tank 2 at time $t > 0$. **Solution:** $y = 200(e^{-t/200} - e^{-t/100})$.
 - (c) What is the maximum amount of corn syrup ever in tank 2? **Solution:** 50
14. Solve $(3x^2 y - 3y) dx + (x^3 - 3x + 2y) dy = 0$ **Solution:** $x^3 y - 2xy + y^2 = C$
15. Solve $y' - y \cos x = 0$ **Solution:** $y = Ce^{\sin x}$

16. For what value of a is $3x^2 + axy + 2 + (6y^2 - x^2 + y)y' = 0$ exact? **Solution:** $a = -2$
17. Solve $y' + y = y^2, y(1) = 1$. **Solution:** $y = 1$ is an equilibrium solution. General solution is $y = 1/(Ce^t + 1)$.
18. Use Euler's method to approximate $y(1.5)$ given $y' = x + \cos(y), y(0) = 2$. Do this with step sizes $h = 0.5$ and $h = 0.25$. **Solution:** With $h = 0.5, y(1.5) \approx 2.25543832$. With $h = 0.25, y(1.5) \approx 3.3527637$. With $h = 0.01, y(1.5) \approx 2.85656$.