Math 217 - September 13, 2010
Solutions

- Substitution: Homogeneous, Bernoulli
- Exact equations

1. Transform the equation with the substitution $v = y/x$.

$$\frac{dy}{dx} = \frac{x^2 + y^2}{2x^2}$$

Solution: $dy/dx = v + x(dv/dx)$.

$$v + x\frac{dv}{dx} = \frac{1 + v^2}{2}$$

2. Transform the equation with the substitution $v = y^{-1}$. (Note both lines below are the same equation, just rearranged.)

$$\begin{aligned} \frac{dy}{dx} + \frac{4}{x}y = & \quad x^3y^2 \\ y^{-2}\frac{dy}{dx} + \frac{4}{x}y^{-1} = & \quad x^3 \end{aligned}$$

Solution: $dv/dx = -y^{-2}\frac{dy}{dx}$.

$$\frac{dv}{dx} + \frac{4}{x}v = x^3$$

3. Solve the previous equations.

   (a)

$$\frac{dy}{dx} = \frac{x^2 + y^2}{2x^2}$$

Solution: $v + x\frac{dv}{dx} = \frac{1+v^2}{2}$ is solved by $-\frac{2}{v-2} = \ln x + C$ so we then have $2x = (x - y)(\ln x + C)$.

   (b)

$$\frac{dy}{dx} + \frac{4}{x} = x^3y^2$$

Solution:

$$\begin{aligned} v = & \quad x^4(C - \ln|x|) \\ \frac{1}{y} = & \quad x^4(C - \ln|x|) \end{aligned}$$

4. Compute the line integral along a curve C from $(1, 2)$ to $(-1, 3)$.

$$\int_{(1,2)}^{(-1,3)} y^3 \, dx + 3xy^2 \, dy = -35$$

Lecture Problems

5. Solve the Bernoulli equation

$$x\frac{dy}{dx} + 5y = 2x^2y^4$$

Solution: Let $v = y^{-3}$ which gives

$$\begin{aligned} v' - \tfrac{15}{x}v &= -6x \\ v &= \tfrac{6}{13}x^2 + Cx^{15} \\ \tfrac{1}{y^3} &= \tfrac{6}{13}x^2 + Cx^{15} \end{aligned}$$

6. Solve the exact equations

(a)

$$(3xy - 3x^2) \, dx + (x^2 - 2y) \, dy = 0$$

Solution: $x^2y - x^3 - y^2 = C$

(b)

$$(y^2 + 1) \, dx + yx \, dy = 0$$

Solution: The equation is NOT exact, trick question. But, the equation obtained by multiplying through by x is exact!!

(c)

$$\cos y \, dx + (y^2 - x \sin y) \, dy = 0$$

Solution: $x \cos y + y^3/3 = C$