

Math 217 - September 9, 2010

Solutions

- Linear equations and applications

1. Solve

$$xy' + 2y = 4x^2; \quad y(1) = 2$$

Solution: General solution: $y = x^2 + \frac{c}{x^2}$. Particular solution: $y = x^2 + \frac{1}{x^2}, x > 0$.

2. Draw a slope field and some solutions curves to

$$xy' + 2y = 4x^2$$

3. Solve

$$x^2y' + xy = \sin x$$

Solution: While an integrating factor can be found which gives

$$xy' + y = \frac{\sin x}{x}$$

Integrating $(\sin x)/x$ must be left like this

$$y = \frac{1}{x} \left(C + \int_1^x \frac{\sin t}{t} dt \right)$$

Lecture Problems

4. A 50 gallon tank initially contains 10 gallons of fresh water. At time $t = 0$ a solution containing 1 lb of salt per gallon is poured into the tank at a rate of 2 gallons per minute. At the same time, the well-stirred mixture leaves the tank at a rate of 4 gallons per minute.

(a) Find an expression for volume of the tank. **Solution:** $V(t) = 10 + 2t$

(b) At what time does the tank overflow? **Solution:** When $t = 20$.

(c) Write down the differential equation to model the amount of salt in the tank. **Solution:**

$$\frac{dx}{dt} = 4 - \frac{2}{10 + 2t}x; \quad x(0) = 0$$

(d) Find an integrating factor for the linear equation. **Solution:** $\rho = 2t + 10$

(e) Find the general solution to the linear equation. **Solution:** $x = \frac{1}{10+2t}(40t + 4t^2 + C)$

(f) Find the particular solution to the linear equation. **Solution:** $x = \frac{1}{10+2t}(40t + 4t^2)$

(g) Find the amount of salt in the tank when the tank overflows. **Solution:** $x(20) = 48$