

Math 217 - September 7, 2010
Solutions

- Separable differential equations
- Logistic equation
- First order linear equations

1. $\frac{dx}{dt} = kx$ (where k is an arbitrary constant) **Solution:** $x = Ae^{kt}$

2. $\frac{dy}{dx} = 3e^y x^2$ **Solution:**

$$e^{-y} dy = 3x^2 dx$$
$$y = -\ln(-x^3 - C)$$

3. $\frac{dy}{dx} = \frac{\sin x}{y \cos y}$ **Solution:**

$$y \cos y dy = \sin x dx$$
$$y = \frac{-\cos x - \cos y + C}{\sin y}$$

4. $\frac{dy}{dx} = -9x^2 y^2$ **Solution:**

$$\frac{dy}{y^2} = -9x^2 dx$$
$$y = \frac{1}{3x^3 + C}$$

5. $\frac{dy}{dx} = -2xe^y$ **Solution:**

$$e^{-y} dy = -2x dx$$
$$y = -\ln(x^2 + C)$$

6. $\frac{dy}{dx} = \frac{4x}{y-1}$ **Solution:**

$$(y-1) dy = 4x dx$$
$$\frac{1}{2}y^2 - y = 2x^2 + C$$

7. $\frac{dy}{dx} = \frac{y^2 - 1}{x} \quad y(1) = 2$ **Solution:**

$$\frac{dy}{y^2 - 1} = \frac{1}{x} dx$$

$$\frac{1}{2} \ln \left(\frac{|y - 1|}{|y + 1|} \right) = \ln |x| + C$$

$$y = \frac{3 + x^2}{3 - x^2}$$

Lecture Problems

8. For the first order linear equation:

- (a) Find the integrating factor: $\rho = e^{\int P(x) dx}$
- (b) Multiply the integrating factor through the differential equation.
- (c) Make sure the left hand side is equal to $(\rho y)'$.
- (d) Integrate the right hand side.
- (e) Solve the differential equation.

$$\frac{dy}{dx} - 2y = 4 - x$$

Solution: $y = -\frac{7}{4} + \frac{1}{2}x + Ce^{2x}$

9. Solve the first order linear equations

- (a) $\frac{dy}{dx} + 3y = 2$ **Solution:** $y = \frac{2}{3} + Ce^{-3x}$
- (b) $\frac{dy}{dx} - 3y = 2$ **Solution:** $y = -\frac{2}{3} + Ce^{3x}$
- (c) $\frac{dy}{dx} - 3y = 2x$ **Solution:** $y = -\frac{2}{9} - \frac{2}{3}x + Ce^{3x}$
- (d) $\frac{dy}{dx} - 3xy = 2x$ **Solution:** $y = -\frac{2}{3} + Ce^{3x^2/2}$