

Math 217 - October 29, 2010
Solutions

• Laplace Transform

$$1. \int_0^{\infty} M e^{ct} e^{-st} dt = \frac{M}{s - c}$$

$$2. \int_0^{\infty} e^{-st} dt = \frac{1}{s}$$

$$3. \int_0^{\infty} e^{-st} e^t dt = \frac{1}{s - 1}$$

$$4. \int_0^{\infty} e^{-st} t dt = \frac{1}{s^2}$$

Lecture Problems

$$5. \quad (a) \quad \mathcal{L}\{2 - 3t + 5t^2 - 13t^3\} = \frac{2}{s} - \frac{3}{s^2} + \frac{10}{s^3} - \frac{78}{s^4}$$

$$(b) \quad \mathcal{L}\left\{\frac{6}{\sqrt{t}} - 7\sqrt{t}\right\} = \frac{6\sqrt{\pi}}{\sqrt{s}} - \frac{7\sqrt{\pi}}{2s^{\frac{3}{2}}}$$

$$(c) \quad \mathcal{L}\{\cosh(at)\} = \mathcal{L}\left\{\frac{e^t + e^{-t}}{2}\right\} = \frac{s}{s^2 - a^2}$$

$$(d) \quad \mathcal{L}\{\sinh(at)\} = \mathcal{L}\left\{\frac{e^t - e^{-t}}{2}\right\} = \frac{a}{s^2 - a^2}$$

$$(e) \quad \mathcal{L}\{\cos at\} = \mathcal{L}\{\cosh(iat)\} = \frac{s}{s^2 + a^2}$$

$$(f) \quad \mathcal{L}\{\sin at\} = \mathcal{L}\left\{\frac{1}{i} \sinh(iat)\right\} = \frac{a}{s^2 + a^2}$$

$$(g) \quad \mathcal{L}\{2 \sin 7t - 5 \sinh 3t\} = \frac{14}{s^2 + 49} - \frac{15}{s^2 - 9}$$

$$6. \quad (a) \quad \mathcal{L}^{-1}\left\{\frac{4}{s - 5}\right\} = 4e^{5t}$$

$$(b) \quad \mathcal{L}^{-1}\left\{\frac{4}{3s - 5}\right\} = \frac{4}{3}e^{5t/3}$$

$$(c) \quad \mathcal{L}^{-1}\left\{\frac{4}{s^2 + 5}\right\} = \frac{4}{\sqrt{5}} \sin(t\sqrt{5})$$