

Math 217 - October 26, 2010

Solutions

- Systems (eigenvalues, eigenvectors)

1. Given the eigenvectors and eigenvalues to matrix A , find the general solution to the equation $X' = AX$. $A = \begin{bmatrix} 1 & 3 \\ -3 & 1 \end{bmatrix}$, $\lambda_1 = 1 - 3i$, $v_1 = [1, -i]^T$, $\lambda_2 = 1 + 3i$, $v_2 = [1, i]^T$.

Solution:

$$X = c_1 \begin{bmatrix} \cos(3t) \\ -\sin(3t) \end{bmatrix} e^t + c_2 \begin{bmatrix} -\sin(3t) \\ -\cos(3t) \end{bmatrix} e^t$$

2. Given the eigenvectors and eigenvalues to matrix A , find the general solution to the equation $X' = AX$. $A = \begin{bmatrix} 1 & -10 \\ 1 & 3 \end{bmatrix}$, $\lambda_1 = 2 - 3i$, $v_1 = [10, -1 + 3i]^T$, $\lambda_2 = 2 + 3i$, $v_2 = [10, -1 - 3i]^T$.

Solution:

$$X = c_1 \begin{bmatrix} 10 \cos(3t) \\ 3 \sin(3t) - \cos(3t) \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} -10 \sin(3t) \\ 3 \cos(3t) + \sin(3t) \end{bmatrix} e^{2t}$$

Lecture Problems

3. Two tanks are filled with salt water and connected by pipes. One pipe pumps brine from tank 1 to tank 2 at a rate of 10 gals/min. Another pipe pumps brine from tank 2 to tank 1 also at a rate of 10 gals/min.

The initial volumes in tanks are 50 gal and 25 gals. The initial amounts of salt in tanks are 15 lbs and none.

Set up (and then solve) the system.

Solution:

$$\begin{aligned} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}' &= \begin{bmatrix} -\frac{1}{5} & \frac{2}{5} \\ \frac{1}{5} & -\frac{2}{5} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \\ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} &= c_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-3t/5} + c_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} \\ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} &= 5 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-3t/5} + 5 \begin{bmatrix} 2 \\ 1 \end{bmatrix} \end{aligned}$$